Topological defects and regularity in non-local gravity

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It is probably necessary to modify General Relativity

The singularities inside General Relativity predict the theory's own breakdown.
Most prominent example: black hole singularities and big bang.

Singularities occur in many classical theories, for example, classical electrodynamics.
Usual solution: taking quantum effects into account removes these singularities.

For gravity, this is not so straightforward.
Different high-energy behavior: the more energy you concentrate, the earlier you form a black hole.

So what can be done?

Here: consider a classical modification of General Relativity that features finite solutions.
In particular: focus on non-local ghost-free gravity.

Modified gravity at the linear level: a framework

Consider quadratic gravity with a modified kinetic structure via the operator $\mathcal{O}^{\mu\nu\rho\sigma}_{\alpha\beta\gamma\delta}$:

$$S = \frac{1}{2}\int \sqrt{-g}\mathrm{d}^D x \left(\frac{1}{\kappa}R + R_{\mu\nu\rho\sigma}\mathcal{O}^{\mu\nu\rho\sigma}_{\alpha\beta\gamma\delta}(\nabla, \Box)R^{\alpha\beta\gamma\delta}\right)$$

$$= \frac{1}{2\kappa}\int \mathrm{d}^D x \Big(\frac{1}{2}h^{\mu\nu}\,\boldsymbol{a}(\Box)\Box\, h_{\mu\nu} - h^{\mu\nu}\,\boldsymbol{a}(\Box)\partial_\mu\partial_\alpha\, h^{\alpha}{}_{\nu} + h^{\mu\nu}\,\boldsymbol{c}(\Box)\partial_\mu\partial_\nu h$$

$$- \frac{1}{2}h\,\boldsymbol{c}(\Box)\Box h + \frac{1}{2}h^{\mu\nu}\,\frac{\boldsymbol{a}(\Box)-\boldsymbol{c}(\Box)}{\Box}\partial_\mu\partial_\nu\partial_\alpha\partial_\beta\, h^{\alpha\beta}\Big)\,.$$

The functions $a(\Box)$ and $c(\Box)$ specify the modification. Different choices:

- Linearized General Relativity: $a(\Box) = c(\Box) = 1$
- Non-local ghost-free gravity: $a(\Box) = c(\Box) = \exp\left[(-\ell^2\Box)^N\right]\,, \quad N = 1, 2, \ldots$

Cosmic strings

Use coordinates $\{t, x, y, z\}$ such that the Minkowski background is $\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$. Cosmic string is sourced by a tension μ via $T_{\mu\nu} = \mu\left(\delta^t_\mu \delta^t_\nu - \delta^z_\mu \delta^z_\nu\right)\delta(x)\delta(y)$.

$$\exp\left(-\ell^2\Box\right)\Box\left(h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu}\right) = -16\pi G T_{\mu\nu}$$

Let us make an ansatz for the metric perturbation $h_{\mu\nu}$ such that

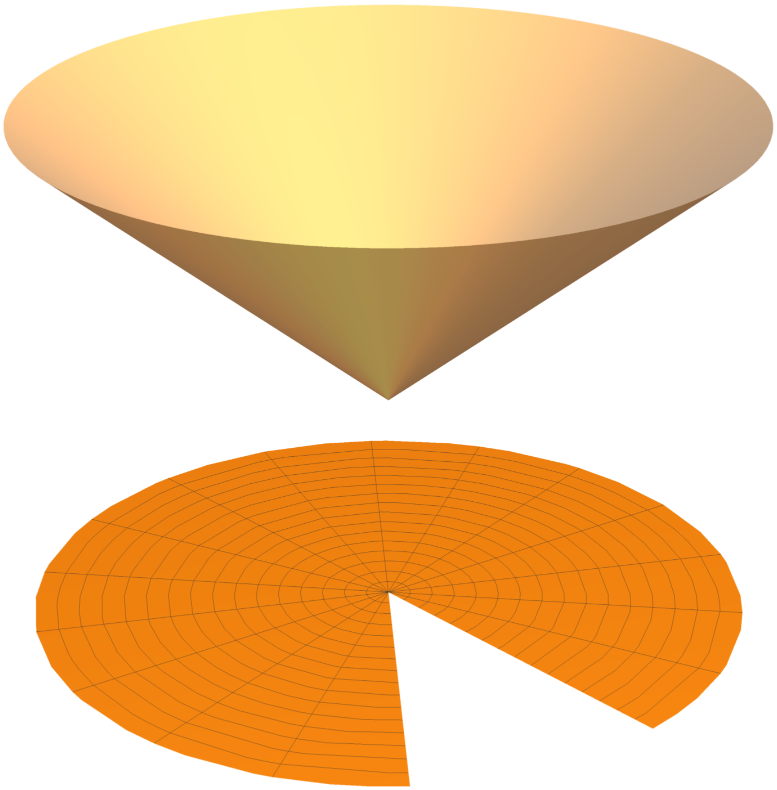
$$h_{\mu\nu}\mathrm{d}X^\mu \mathrm{d}X^\nu = \psi(\rho)\left[\mathrm{d}t^2 + \mathrm{d}z^2\right] + \phi(\rho)\left[\mathrm{d}x^2 + \mathrm{d}y^2\right], \quad \rho^2 \equiv x^2 + y^2 .$$

The field equation is effectively a 2D problem, and it is solved by

$$\psi = 0\,, \quad \phi(\rho) = 4G\mu\left[\mathrm{Ei}\left(-\tfrac{\rho^2}{4\ell^2}\right) - 2\ln\left(\tfrac{\rho}{\rho_0}\right)\right] .$$

For $\rho/\ell \to \infty$ one recovers the cosmic string of General Relativity.

Properties of the cosmic string. Topology and regularity.



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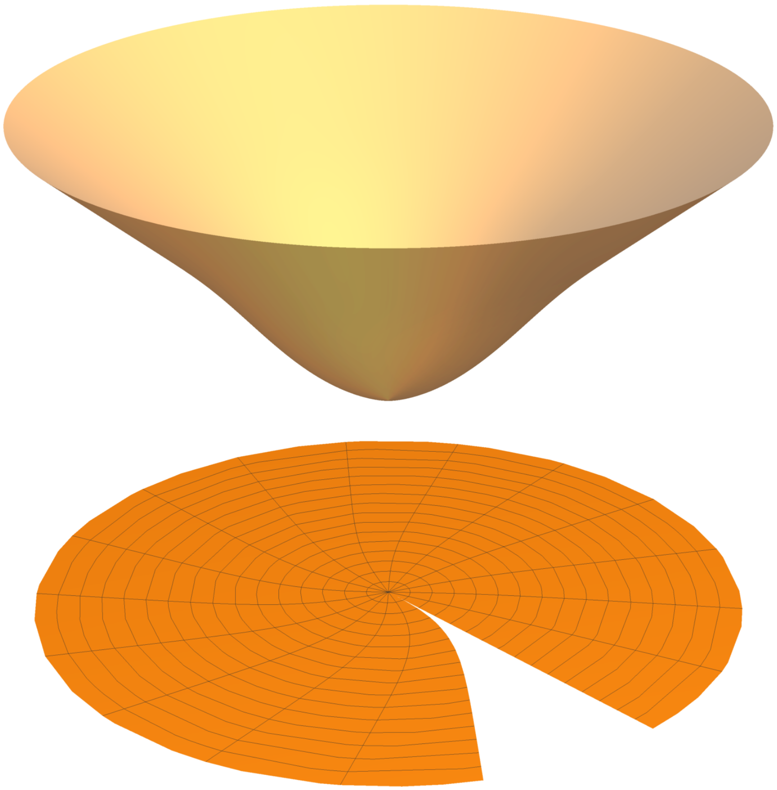

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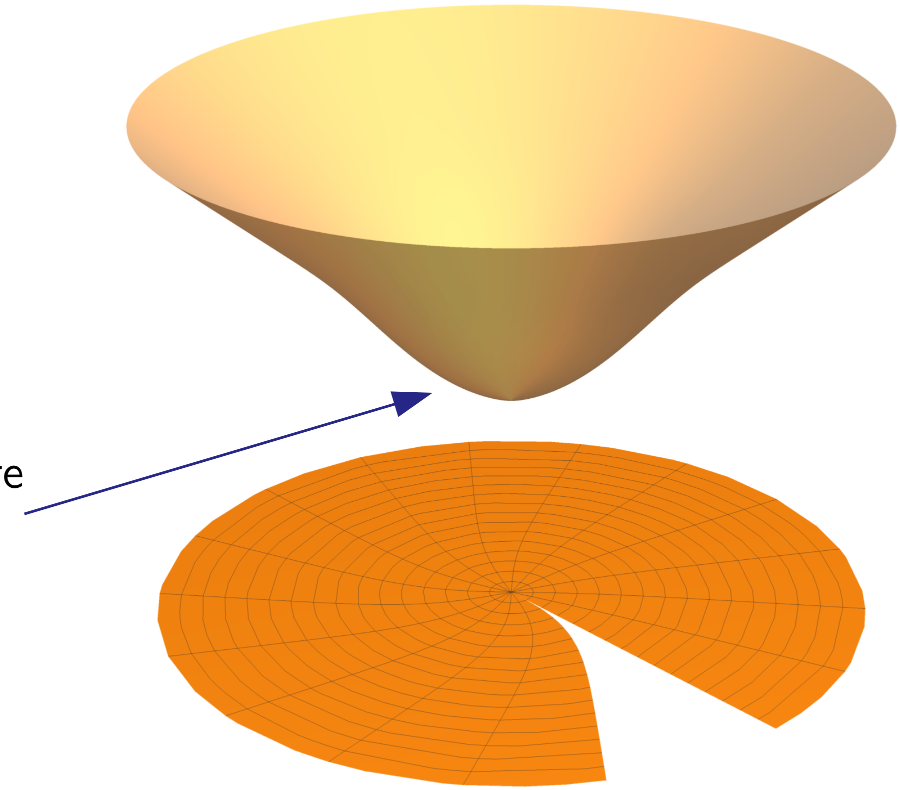

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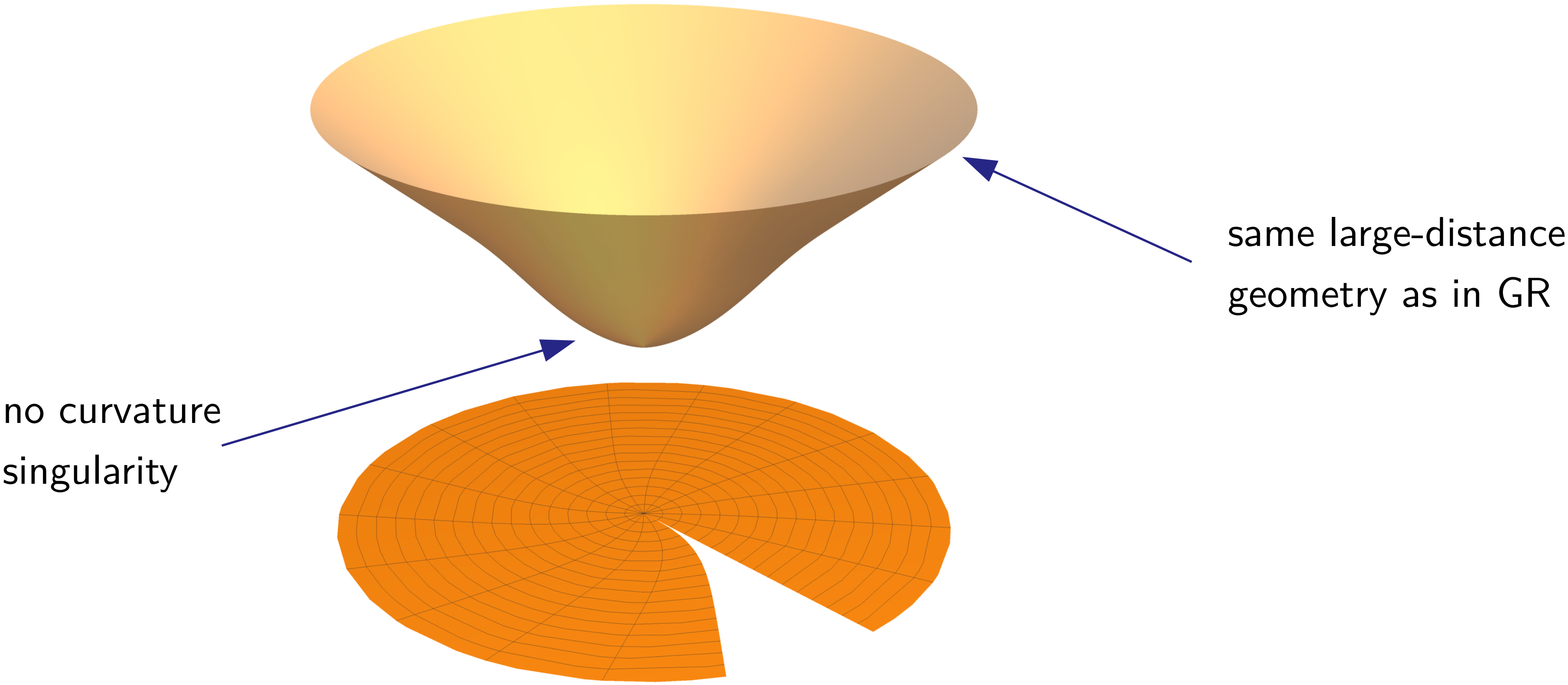

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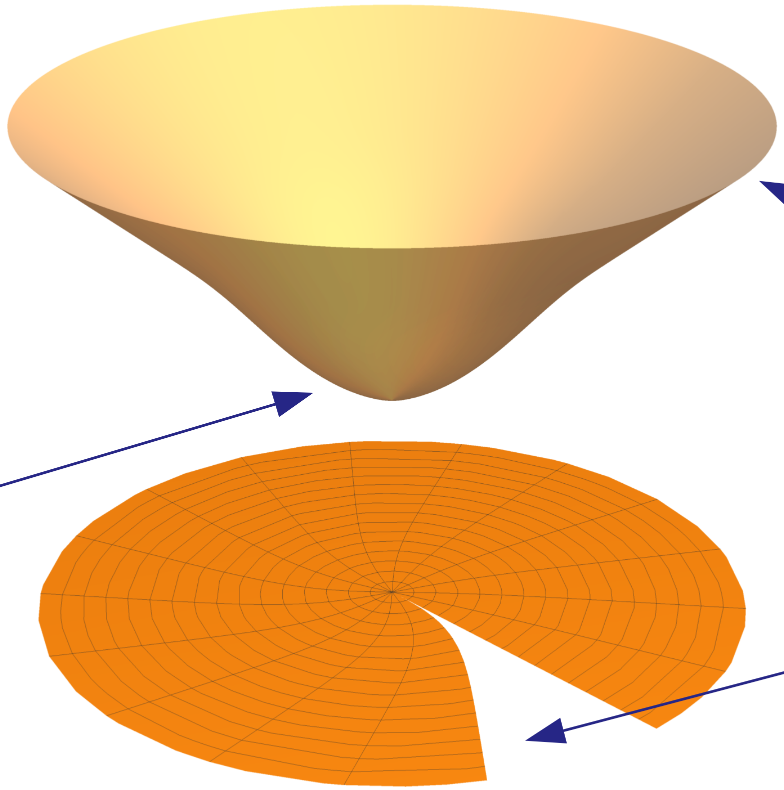


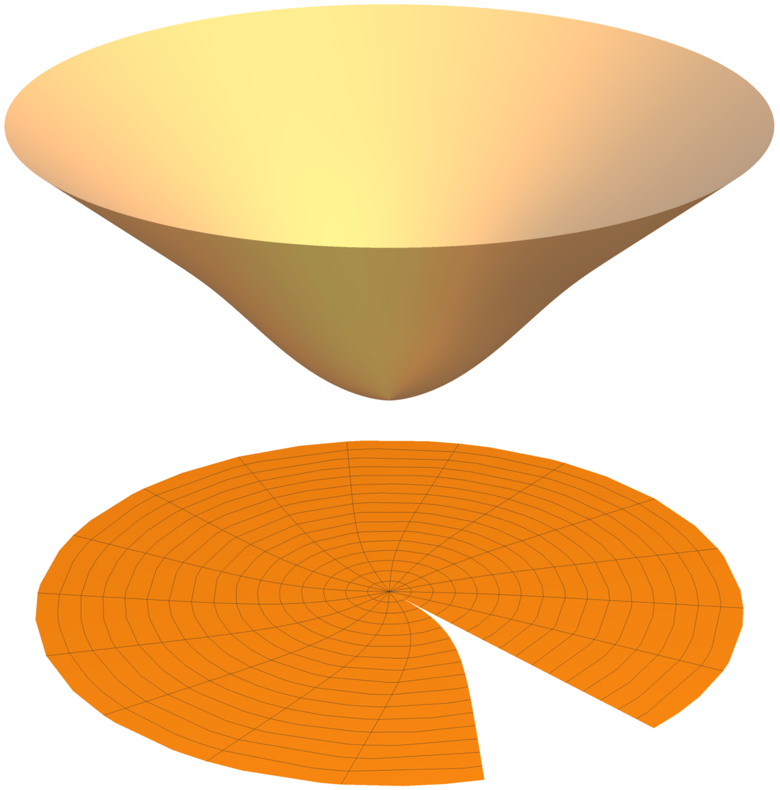
same large-distance geometry as in GR

no curvature singularity

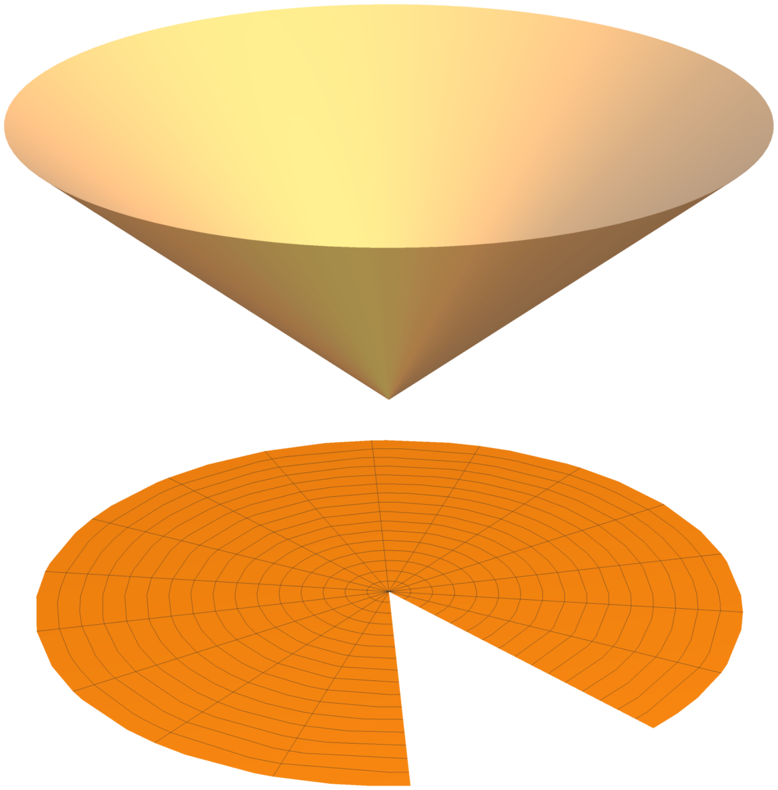
angle deficit is function of radius!

$$\delta\varphi = 8\pi G\mu \left[1 - \frac{\sqrt{\pi}\ell}{\rho}\mathrm{erf}\left(\frac{\rho}{2\ell}\right)\right]$$

Comparison



non-local gravity



General Relativity

Summary & conclusions

- One can define a linearized non-local theory of gravity that is devoid of singularities.
  "Non-local infinite-derivative ghost-free gravity."

- At present, not much is known about exact solutions, except in a few cases.

- Common wisdom: topological properties = global properties.
- Do these regular theories challenge that notion? Can there even be defects anymore?
- And can these quasi-topological quantities be observed in Nature?

- It is conceivable that similar concepts arise in regular theories, applied to either gravity or simple quantum systems (think geometric phases).

Thank you for your attention.

Abstract

Topological defects and regularity in non-local gravity

Non-local gravitational theories with infinitely many derivatives may solve the gravitational singularity problem without introducing ghostlike degrees of freedom that one typically encounters in higher-order gravity. However, due to the complexity of the non-linear non-local field equations, so far only the linear regime is understood well. In this talk I will focus on cosmic string solutions obtained in weak-field non-local gravity. These have an interesting feature: non-locality regularizes the curvature defect at the location of the cosmic string. Since spacetime is now simply connected one might assume that the angle deficit vanishes, but this is not true: asymptotically one recovers the string solution of General Relativity. Non-locality hence challenges the way we think about topological defects in connection with topological properties of spacetime. If time permits, I shall also comment on similar effects regarding Aharanov–Bohm phases in non-local quantum mechanics, and their possible observational signatures.

Jens Boos, “Angle deficit & non-local gravitoelectromagnetism around a slowly spinning cosmic string,”
Int. J. Mod. Phys. D **29** (2020) no. 14, 2043027; arXiv:2003.13847 [gr-qc],
honorable mention in the Gravity Research Foundation Essay Competition 2020.