Unexpected features of non-locality: resonant particle production

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Based on...

Resonant particle creation by a time-dependent potential in a nonlocal theory

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Considering an exactly solvable local quantum theory of a scalar field interacting with a δ-shaped time-dependent potential we calculate the Bogoliubov coefficients analytically and determine the spectrum of created particles. We then show how these considerations, when suitably generalized to a specific nonlocal "infinite-derivative" quantum theory, are impacted by the presence of nonlocality. In this model, nonlocality leads to a significant resonant amplification of certain modes, leaving its imprint not only in the particle spectrum but also in the total number density of created particles.

arXiv:2011.12929 [hep-th], submitted to Physics Letters B.

More work on non-local theories with Valeri Frolov and Andrei Zelnikov.

Motivation

Non-locality plays an important role in modern physics:

- Entanglement is a non-local phenomenon in quantum theory.
- Many effective actions in quantum field theory are non-local.
- It is probably impossible to define local observables in quantum gravity.
- Non-locality may solve the black hole information loss problem.

But: non-locality also challenges many of our "standard" notions in theoretical physics.

- Causality is typically violated at some scale (and perhaps beyond).
- Variational principle is not necessarily self-consistent.
- Notion of a "local particle" is difficult to define.

Solution: discuss reasonably well-behaved non-local theories, and use asymptotic local properties.

Unexpected features of non-locality: resonant particle production

- quantum theory of a scalar field
- particle creation by time-dependent potentials
- exactly solvable non-local model

PART I

QUANTUM THEORY OF A SCALAR FIELD

creation and annihilation operators ▪ definition of vacuum

Classical scalar field

A free classical scalar field in Minkowski spacetime:

$$\Box\varphi(t,\boldsymbol{x}) = 0\,, \qquad \Box = -\partial_t^2 + \nabla^2$$

We don't know any real scalar fields in Nature, use it as a toy model.

Use a Fourier transform to express a solution of this field equation:

$$\varphi(t,\boldsymbol{x}) = \int \frac{\mathrm{d}^d k}{(2\pi)^{d/2}} \frac{1}{\sqrt{2\omega_{\boldsymbol{k}}}} \Big[\, e^{-i\omega_{\boldsymbol{k}} t + i\boldsymbol{k}\cdot\boldsymbol{x}}\, a_{\boldsymbol{k}} \;+\; e^{+i\omega_{\boldsymbol{k}} t - i\boldsymbol{k}\cdot\boldsymbol{x}}\, a^*_{\boldsymbol{k}} \,\Big]$$

The frequency $\omega_{\boldsymbol{k}}$ is given by the dispersion relation $\omega_{\boldsymbol{k}}^2 = \boldsymbol{k}^2$.

The coefficients $a_{\boldsymbol{k}}$ are the (complex) Fourier components of the field φ and satisfy $a_{-\boldsymbol{k}} = a^*_{\boldsymbol{k}}$.

So how can we make this a quantum theory?

Quantum scalar field

A free **quantum** scalar field in Minkowski spacetime:

$$\Box\hat{\varphi}(t,\boldsymbol{x}) = 0\,, \qquad \Box = -\partial_t^2 + \nabla^2$$

This equation is a differential equation for a "field operator."

Use a Fourier transform to express a solution of this field equation:

$$\hat{\varphi}(t,\boldsymbol{x}) = \int \frac{\mathrm{d}^d k}{(2\pi)^{d/2}} \frac{1}{\sqrt{2\omega_{\boldsymbol{k}}}} \Big[\, e^{-i\omega_{\boldsymbol{k}} t + i\boldsymbol{k}\cdot\boldsymbol{x}}\, \hat{a}_{\boldsymbol{k}} \; + \; e^{+i\omega_{\boldsymbol{k}} t - i\boldsymbol{k}\cdot\boldsymbol{x}}\, \hat{a}^\dagger_{\boldsymbol{k}} \,\Big]$$

The frequency $\omega_{\boldsymbol{k}}$ is given by the dispersion relation $\omega_{\boldsymbol{k}}^2 = \boldsymbol{k}^2$.

The operators $\hat{a}^\dagger_{\boldsymbol{k}}$ and $\hat{a}_{\boldsymbol{k}}$ are so-called creation and annihilation operators of the field $\hat{\varphi}$.

They can act on the vacuum state $|0\rangle$ to create or remove quanta of momentum $\boldsymbol{k}$.

What is the main idea of these operators?

$$|\Psi\rangle = |m_{\boldsymbol{k}},\ n_{\boldsymbol{q}},\ \ldots\rangle$$

state of the
entire system

number of particles
with momentum $\boldsymbol{k}$,
add one with $\hat{a}^{\dagger}_{\boldsymbol{k}}$,
remove one with $\hat{a}_{\boldsymbol{k}}$.

number of particles
with momentum $\boldsymbol{q}$,
add one with $\hat{a}^{\dagger}_{\boldsymbol{q}}$,
remove one with $\hat{a}_{\boldsymbol{q}}$.

Some more basics on creation and annihilation operators

Definition of the vacuum state:

$$\hat{a}_{\boldsymbol{k}}|0\rangle = 0$$

Commutation relations (needed for canonical commutation relations at the Hamiltonian level):

$$[\hat{a}_{\boldsymbol{k}}, \hat{a}^{\dagger}_{\boldsymbol{k}'}] = \delta^{(d)}(\boldsymbol{k} - \boldsymbol{k}')\,, \qquad [\hat{a}_{\boldsymbol{k}}, \hat{a}_{\boldsymbol{k}'}] = [\hat{a}^{\dagger}_{\boldsymbol{k}}, \hat{a}^{\dagger}_{\boldsymbol{k}'}] = 0$$

The concrete representation of $\hat{a}^{\dagger}_{\boldsymbol{k}}$ and $\hat{a}_{\boldsymbol{k}}$ is not so important. Use basic relations:

$$\hat{a}^{\dagger}_{\boldsymbol{k}}|n_{\boldsymbol{k}}\rangle = \sqrt{n_{\boldsymbol{k}} + 1}|n_{\boldsymbol{k}} + 1\rangle\,, \quad \hat{a}_{\boldsymbol{k}}|n_{\boldsymbol{k}}\rangle = \sqrt{n_{\boldsymbol{k}}}|n_{\boldsymbol{k}} - 1\rangle$$

Then, many-particle states can be written like this:

$$|m_{\boldsymbol{k}}, n_{\boldsymbol{q}}, \ldots\rangle \;\sim\; (\hat{a}^{\dagger}_{\boldsymbol{k}})^m|0\rangle \otimes (\hat{a}^{\dagger}_{\boldsymbol{q}})^n|0\rangle \otimes \ldots$$

You can count particles with the occupation number density operator $\hat{n}_{\boldsymbol{k}} = \hat{a}^{\dagger}_{\boldsymbol{k}}\hat{a}_{\boldsymbol{k}}$.

quantum theory
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potentials

exactly solvable
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PART II

PARTICLE CREATION BY POTENTIALS

potentials ▪ Bogoliubov coefficients ▪ Lippmann–Schwinger equation

Quantum scalar field with time-dependent potential

A quantum scalar field in Minkowski spacetime with **time-dependent potential**:

$$\left[\Box - V(t)\right] \hat{\varphi}(t, \boldsymbol{x}) = 0\,, \qquad \Box = -\partial_t^2 + \nabla^2$$

This equation is again a differential equation for the "field operator" $\hat{\varphi}(t, \boldsymbol{x})$.

Use a purely spatial Fourier transform to rewrite a solution of this field equation:

$$\hat{\varphi}(t, \boldsymbol{x}) = \int \frac{\mathrm{d}^d k}{(2\pi)^{d/2}} \left[\, e^{+i\boldsymbol{k}\cdot\boldsymbol{x}}\, \hat{\varphi}_{\boldsymbol{k}}(t) \; + \; e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\, \hat{\varphi}^\dagger_{\boldsymbol{k}}(t) \,\right]$$

Can now study the equation for the operators $\hat{\varphi}_{\boldsymbol{k}}(t)$ directly:

$$\left[\partial_t^2 + \omega_{\boldsymbol{k}}^2 + V(t)\right] \hat{\varphi}_{\boldsymbol{k}}(t) = 0$$

This is still a very complicated problem.

Quantum scalar field with time-dependent potential

Let's assume that the potential vanishes at early and late times, $\left(\partial_t^2 + \omega_{\boldsymbol{k}}^2\right)\hat{\varphi}_{\boldsymbol{k}}(t) = 0$.
At those "asymptotic times" there is no potential term and have again the free solution from before:

$$\hat{\varphi}_{\boldsymbol{k}}(t \to -\infty) = \frac{e^{-i\omega_{\boldsymbol{k}} t}}{\sqrt{2\omega_{\boldsymbol{k}}}}\hat{a}_{\boldsymbol{k}}\,, \quad \hat{\varphi}_{\boldsymbol{k}}(t \to +\infty) = \frac{e^{-i\omega_{\boldsymbol{k}} t}}{\sqrt{2\omega_{\boldsymbol{k}}}}\hat{b}_{\boldsymbol{k}} + \frac{e^{+i\omega_{\boldsymbol{k}} t}}{\sqrt{2\omega_{\boldsymbol{k}}}}\hat{b}^\dagger_{\boldsymbol{k}}$$

Note the different creation and annihilation operators at **early times** and at **late times**.
There are two different vacua ("in"-vacuum and "out"-vacuum):

$$\hat{a}_{\boldsymbol{k}}|\mathrm{in};0\rangle = 0\,, \quad \hat{b}_{\boldsymbol{k}}|\mathrm{out};0\rangle = 0$$

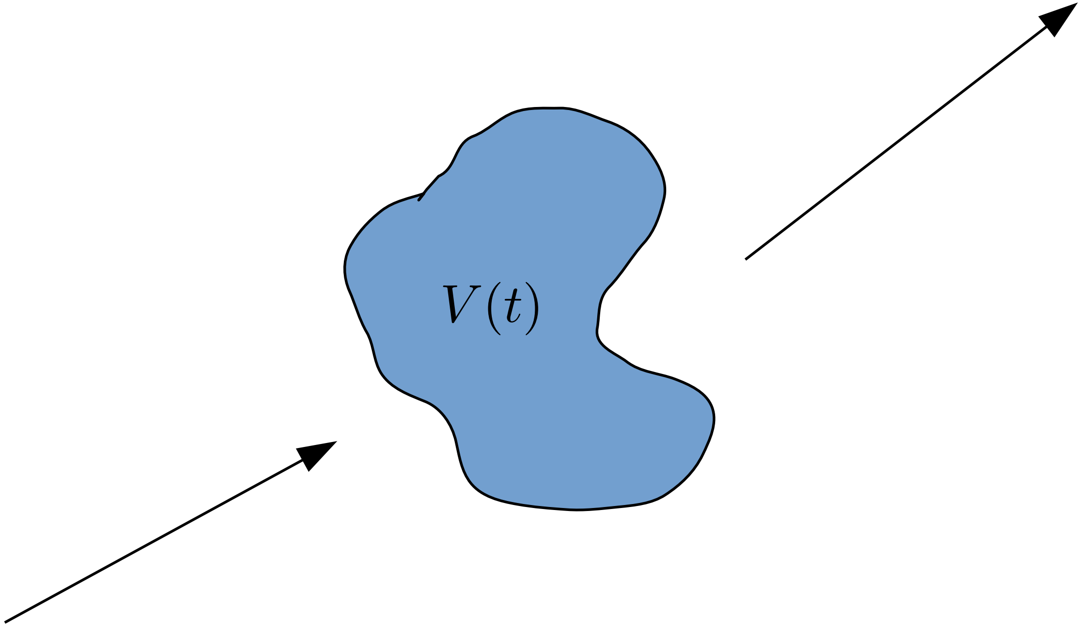
Bogoliubov coefficients relate these operators ($|\alpha_{\omega_{\boldsymbol{k}}}|^2 - |\beta_{\omega_{\boldsymbol{k}}}|^2 = 1$ ensures canonical trafo):

$$\hat{b}_{\boldsymbol{k}} = \alpha_{\omega_{\boldsymbol{k}}}\hat{a}_{\boldsymbol{k}} + \beta^*_{\omega_{\boldsymbol{k}}}\hat{a}^\dagger_{-\boldsymbol{k}}\,, \quad \hat{b}^\dagger_{\boldsymbol{k}} = \alpha^*_{\omega_{\boldsymbol{k}}}\hat{a}^\dagger_{\boldsymbol{k}} + \beta_{\omega_{\boldsymbol{k}}}\hat{a}_{-\boldsymbol{k}}$$

And what does all of this have to do with particle creation?

$$n_{\boldsymbol{k}} = \langle 0|\hat{b}^\dagger_{\boldsymbol{k}}\hat{b}_{\boldsymbol{k}}|0\rangle$$
$$= |\beta_{\omega_{\boldsymbol{k}}}|^2 \neq 0$$

asymptotic future



asymptotic past

$$\hat{a}_{\boldsymbol{k}}|0\rangle = 0$$

$$n_{\boldsymbol{k}} = \langle 0|\hat{a}^\dagger_{\boldsymbol{k}}\hat{a}_{\boldsymbol{k}}|0\rangle = 0$$

Main insight: can extract this factor solely from the classical late-time asymptotics!

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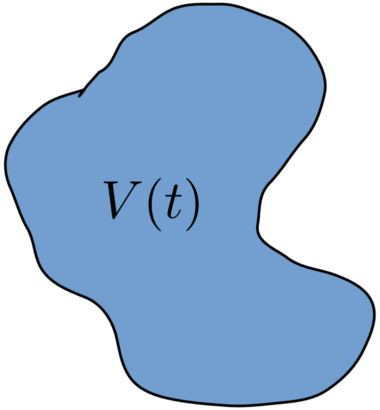

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Main insight: can extract this factor solely from the classical late-time asymptotics!

Lippmann–Schwinger equation

How to extract the late-time asymptotics? Use the retarded Green function:

$$(\partial_t^2 + \omega_{\boldsymbol{k}}^2)\varphi_k(t) = -V(t)\varphi_k(t)$$

$$(\partial_t^2 + \omega_{\boldsymbol{k}}^2)G_{\boldsymbol{k}}^{\mathrm{R}}(t'-t) = -\delta(t'-t)\,, \quad G_{\boldsymbol{k}}^{\mathrm{R}}(t'-t) = 0 \text{ if } t' < t\,.$$

Then, the solution can be written in Lippmann–Schwinger form like this:

$$\varphi_{\boldsymbol{k}}(t) = \varphi_{\boldsymbol{k}}^0(t) + \int_{-\infty}^{\infty} \mathrm{d}t' G_{\boldsymbol{k}}^{\mathrm{R}}(t-t')V(t')\varphi_{\boldsymbol{k}}(t')$$

Important: $\varphi_{\boldsymbol{k}}^0$ is a free solution that encodes the asymptotic past (because $G_{\boldsymbol{k}}^{\mathrm{R}}$ vanishes there).

$$\varphi_{\boldsymbol{k}}^0(t) = \frac{1}{\sqrt{2\omega_{\boldsymbol{k}}}} e^{-i\omega_{\boldsymbol{k}} t}$$

Lippmann–Schwinger equation

This integral collapses for a delta-shaped potential $V(t) = \lambda\delta(t)$.

This potential also vanishes at early and late times, so we can expect a reasonable result.

$$\varphi_{\boldsymbol{k}}(t) = \varphi^0_{\boldsymbol{k}}(t) + \int\limits_{-\infty}^{\infty} \mathrm{d}t' G^{\mathrm{R}}_{\boldsymbol{k}}(t-t')V(t')\varphi_{\boldsymbol{k}}(t') = \varphi^0_{\boldsymbol{k}}(t) + \lambda G^{\mathrm{R}}_{\boldsymbol{k}}(t)\varphi_{\boldsymbol{k}}(0)$$

$$\varphi_{\boldsymbol{k}}(0) = \frac{\varphi^0_{\boldsymbol{k}}(0)}{1-\lambda G^{\mathrm{R}}_{\boldsymbol{k}}(0)}$$

The local retarded Green function has this form (sum of positive and negative frequencies):

$$G^{\mathrm{R}}_{\boldsymbol{k}}(t'-t) = \frac{i}{2\omega_{\boldsymbol{k}}}\left[e^{+i\omega_{\boldsymbol{k}}(t'-t)} - e^{-i\omega_{\boldsymbol{k}}(t'-t)}\right]\theta(t'-t)$$

Extracting the Bogoliubov coefficient

Collecting all these steps we find the following solution in the future:

$$\varphi_{\boldsymbol{k}}(t) = \left(1 - \frac{i\lambda}{2\omega_{\boldsymbol{k}}}\right) \frac{e^{-i\omega_{\boldsymbol{k}} t}}{\sqrt{2\omega_{\boldsymbol{k}}}} + \frac{i\lambda}{2\omega_{\boldsymbol{k}}} \frac{e^{+i\omega_{\boldsymbol{k}} t}}{\sqrt{2\omega_{\boldsymbol{k}}}}$$

Read off the negative frequency components which correspond to out-particles, $|\beta_{\omega_{\boldsymbol{k}}}|^2 = \dfrac{\lambda^2}{4\omega_{\boldsymbol{k}}^2}$.
This means that there is a non-zero particle number in the asymptotic future.

So, as a recipe:

1. Construct exact solution in presence of potential via Lippmann–Schwinger.
2. Make sure the free solution encodes the correct in-vacuum.
3. Compute the late-time asymptotics.
4. Read off the **Bogoliubov coefficient**.

But the result is not terribly exciting and could be guessed via dimensional analysis.

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PART III

EXACTLY SOLVABLE NON-LOCAL MODEL

non-local Green function ▪ resonant particle creation

Non-local model

Let's consider instead this **non-local** model:

$$\exp\left[\ell^4(\partial_t^2+\omega_{\boldsymbol{k}}^2)^2\right](\partial_t^2+\omega_{\boldsymbol{k}}^2)\varphi_k(t) = -V(t)\varphi_k(t)$$

The non-local retarded Green function satisfies

$$\exp\left[\ell^4(\partial_t^2+\omega_{\boldsymbol{k}}^2)^2\right](\partial_t^2+\omega_{\boldsymbol{k}}^2)\mathcal{G}_{\boldsymbol{k}}^{\mathrm{R}}(t'-t) = -\delta(t'-t)\,.$$

It has the following important property (DeWitt's "asymptotic causality"):

$$\mathcal{G}_{\boldsymbol{k}}^{\mathrm{R}}(t'-t\pm\infty) = G_{\boldsymbol{k}}^{\mathrm{R}}(t'-t)$$

This means that at early and late times physics is unchanged. **Or is it?**

Non-local model: surprising result!

Performing the same calculations again we find for the particle creation rate:

$$\beta_{\omega_{\boldsymbol{k}}} = \frac{i\lambda}{2\omega_{\boldsymbol{k}}} \frac{1}{1 - \lambda \mathcal{G}^{\mathrm{R}}_{\boldsymbol{k}}(0)}$$

Observation: there is a new effect solely due to non-locality!

$$\mathcal{G}^{\mathrm{R}}_{\boldsymbol{k}}(0) = \frac{\Gamma\left(\frac{3}{4}\right)\ell}{\pi} {}_2F_2\left(\tfrac{1}{4}, \tfrac{3}{4}; \tfrac{1}{2}, \tfrac{5}{4}; -k^4\ell^4\right) - \frac{\sqrt{2}k^2\ell^3}{6\Gamma\left(\frac{3}{4}\right)} {}_2F_2\left(\tfrac{3}{4}, \tfrac{5}{4}; \tfrac{3}{2}, \tfrac{7}{4}; -k^4\ell^4\right) \quad \neq 0$$

This means that there exists a critical wave number for which the **particle creation rate diverges**!
(Provided the potential is positive and above a critical threshold: $\lambda\ell > \lambda_\star\ell = \pi/\Gamma\left(\frac{3}{4}\right) \approx 2.46369...$)

This effect is quite universal in this class of non-local theories, since it only assumes asymptotic causality. The behavior of the non-local Green function at $t = 0$ dictates the creation rate.

PART IV

CONCLUSIONS AND OUTLOOK

congratulations ▪ we ▪ made ▪ it

Conclusions and outlook

We found an **unexpected resonant particle creation** due to non-locality.

- This effect is "non-perturbative" since it is an exact solution.
- It arises solely under the assumption of asymptotic causality.
- The scale of non-locality $\ell > 0$ introduces some sort of resonance since it smears out sharp delta-shaped objects.

Open questions/future directions:

- Does this happen for other potentials?
- Could it have implications for cosmology?
- What do you think?

Thank you for your attention :)

Abstract

Unexpected features of non-locality: resonant particle production

Let's consider a linear scalar field theory in the presence of an impulsive potential delta(t), which is an exactly solvable model. If there is a quantum mechanical vacuum at early times, then the potential term at t=0 creates a non-zero particle number at late times far into future. This means that the vacuum state of early times is mapped into a non-vacuum state at late times, and this can be described by so-called Bogoliubov coefficients in the framework of second quantization/Fock space quantization. In this talk I will extend these studies to an exactly solvable non-local model and explain how the future particle spectrum is impacted by the presence of non-locality. Surprisingly, there appears a strong resonant amplification of certain modes, leading to a burst of particles at late times.

Based on: Jens Boos, Valeri P. Frolov, and Andrei Zelnikov, "Resonant particle creation by a time-dependent potential in a nonlocal theory," arXiv:2011.12929 [hep-th], submitted to Physics Letters B.