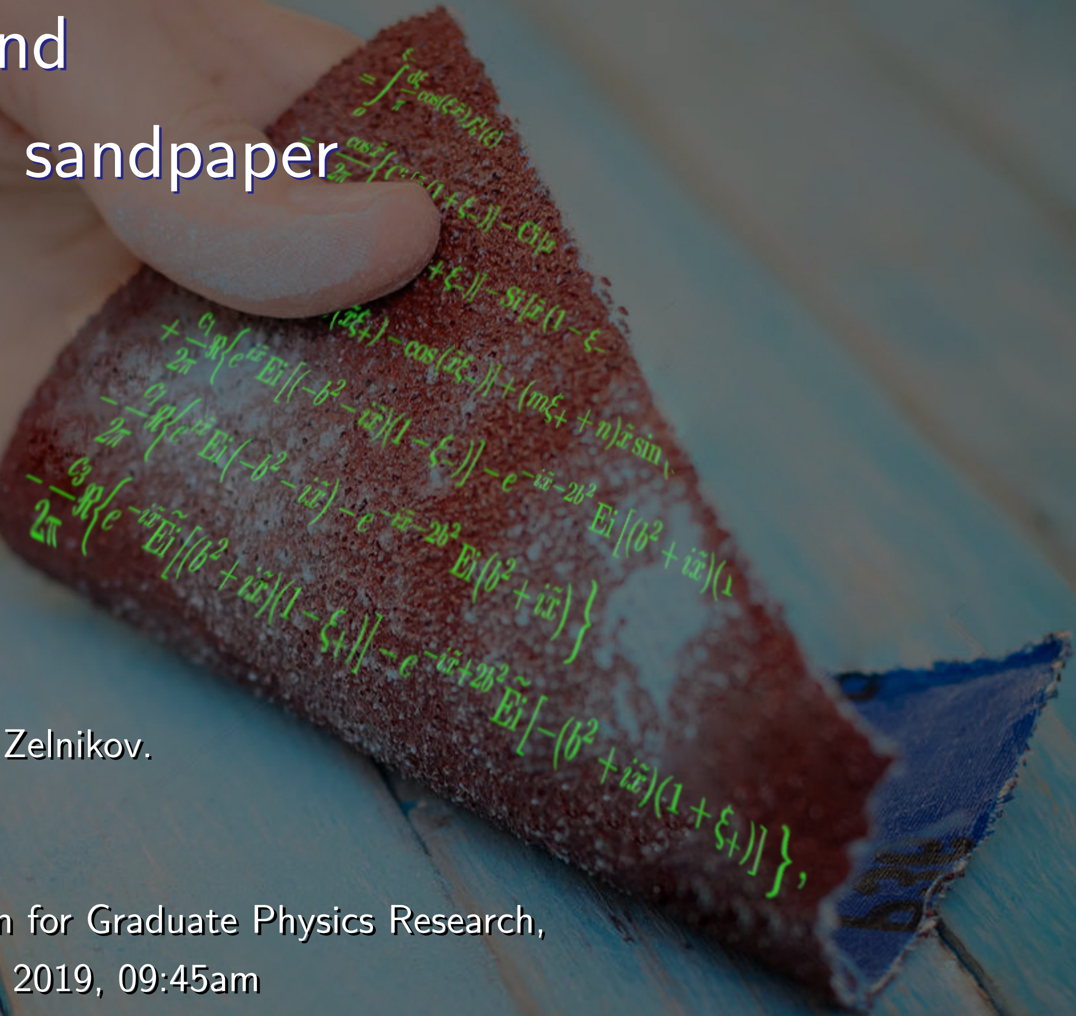
Black holes and mathematical sandpaper



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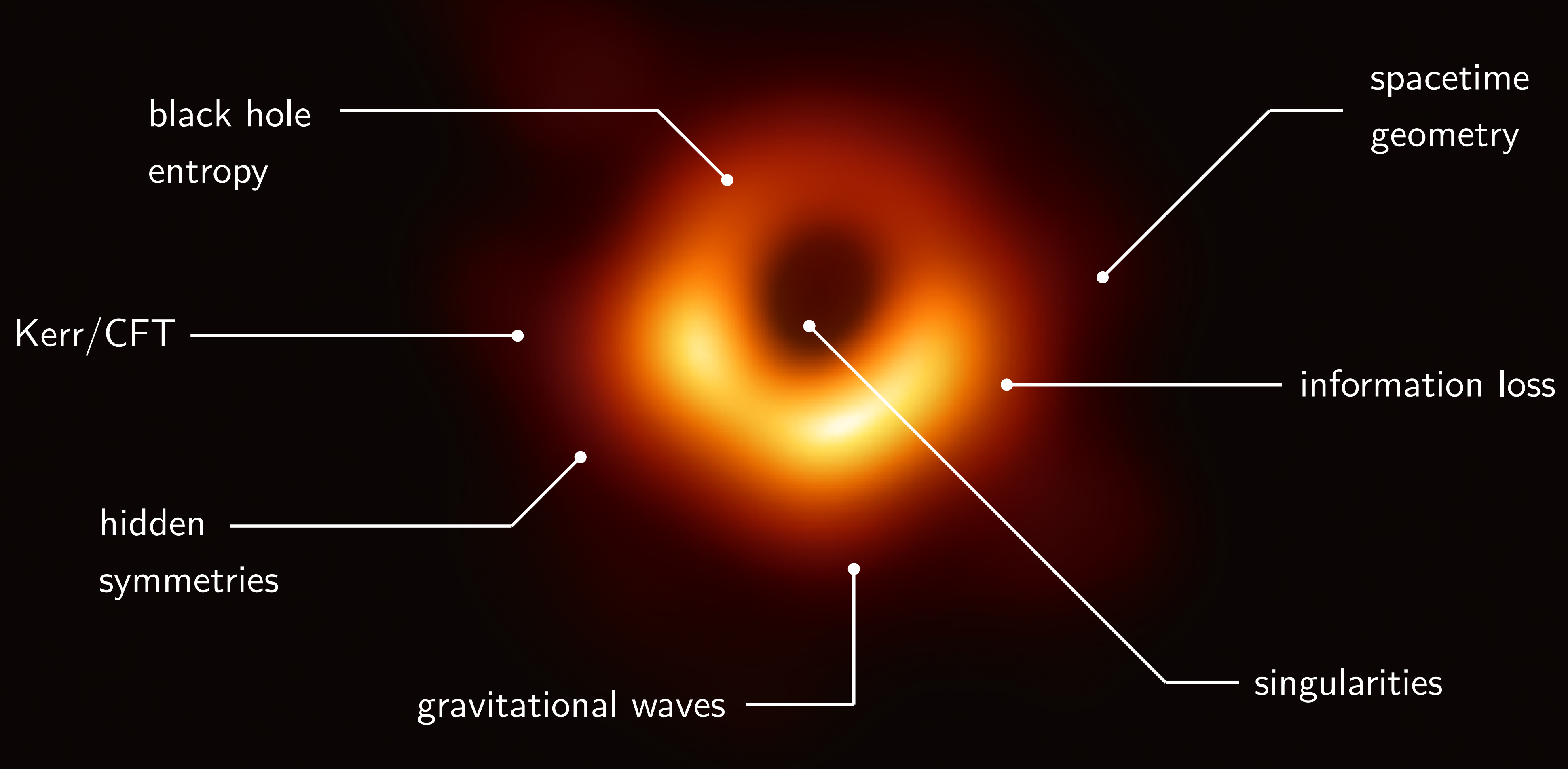
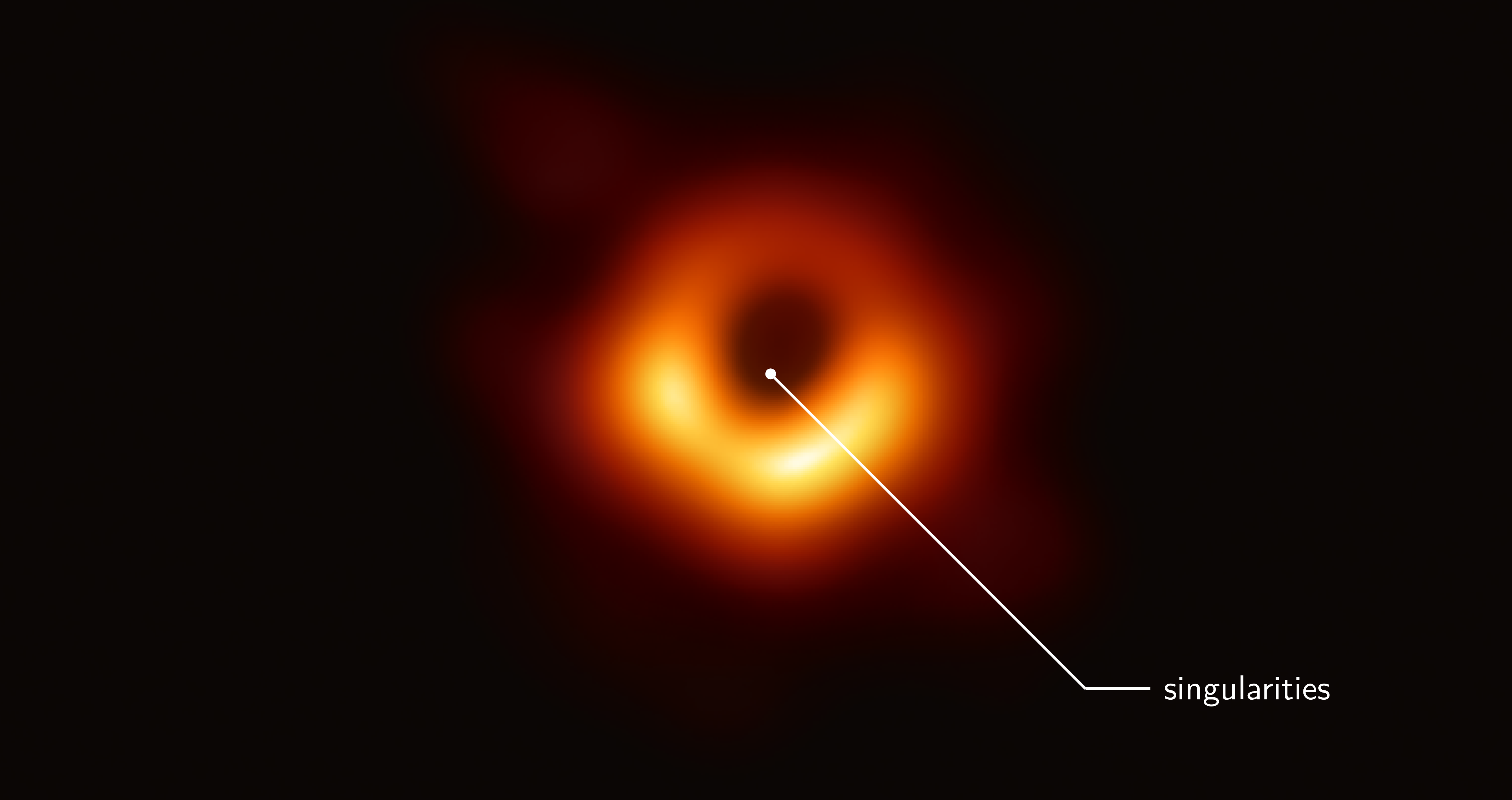
Joint work with
Valeri Frolov and Andrei Zelnikov.

Tenth Annual Symposium for Graduate Physics Research,
Thursday, September 26, 2019, 09:45am

Rotating black holes as a testing ground for new physics





April 10, 2019, Event Horizon Telescope Collaboration

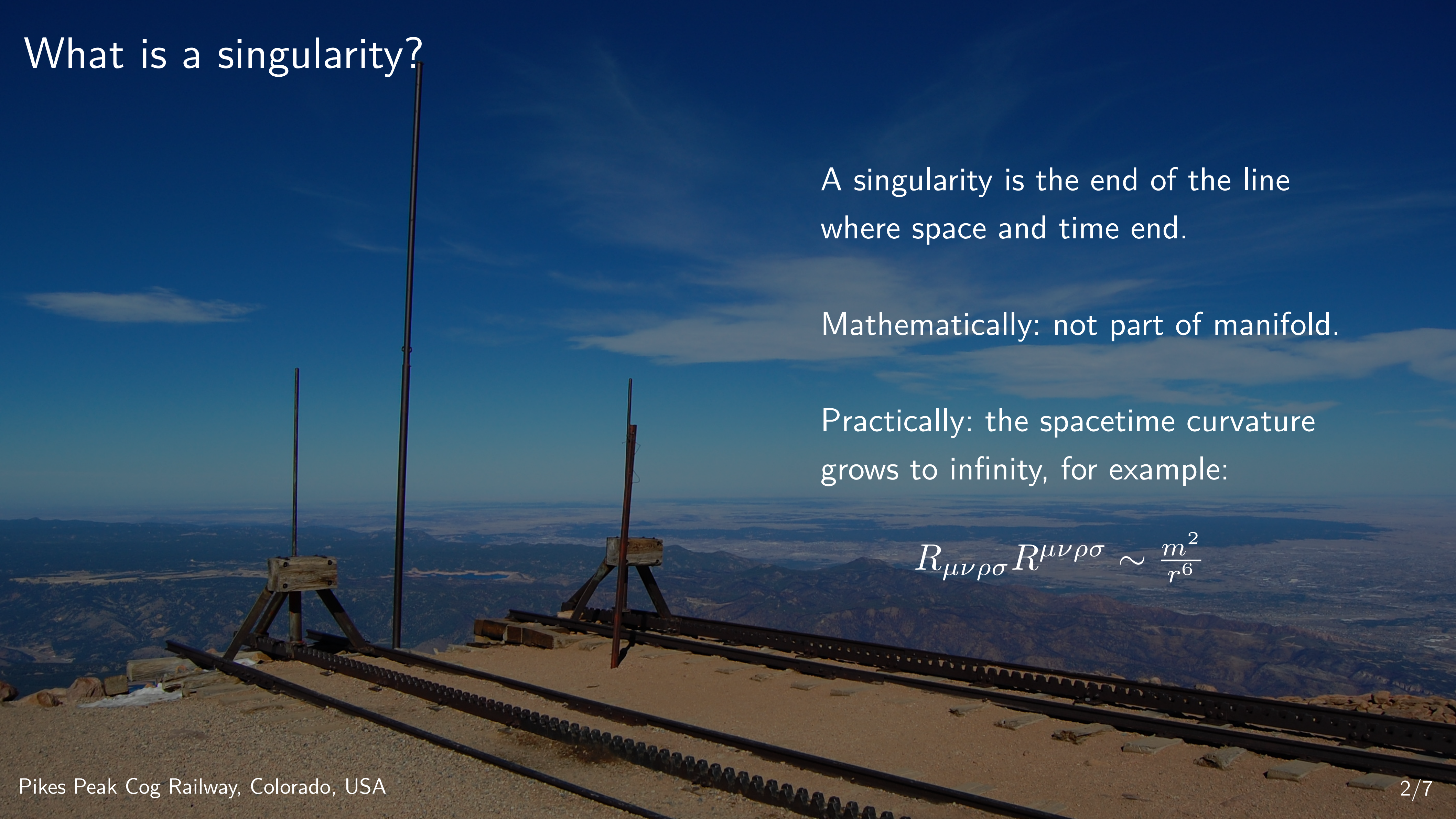
What is a singularity?

A singularity is the end of the line where space and time end.

Mathematically: not part of manifold.

Practically: the spacetime curvature grows to infinity, for example:

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \sim \frac{m^2}{r^6}$$



Pikes Peak Cog Railway, Colorado, USA

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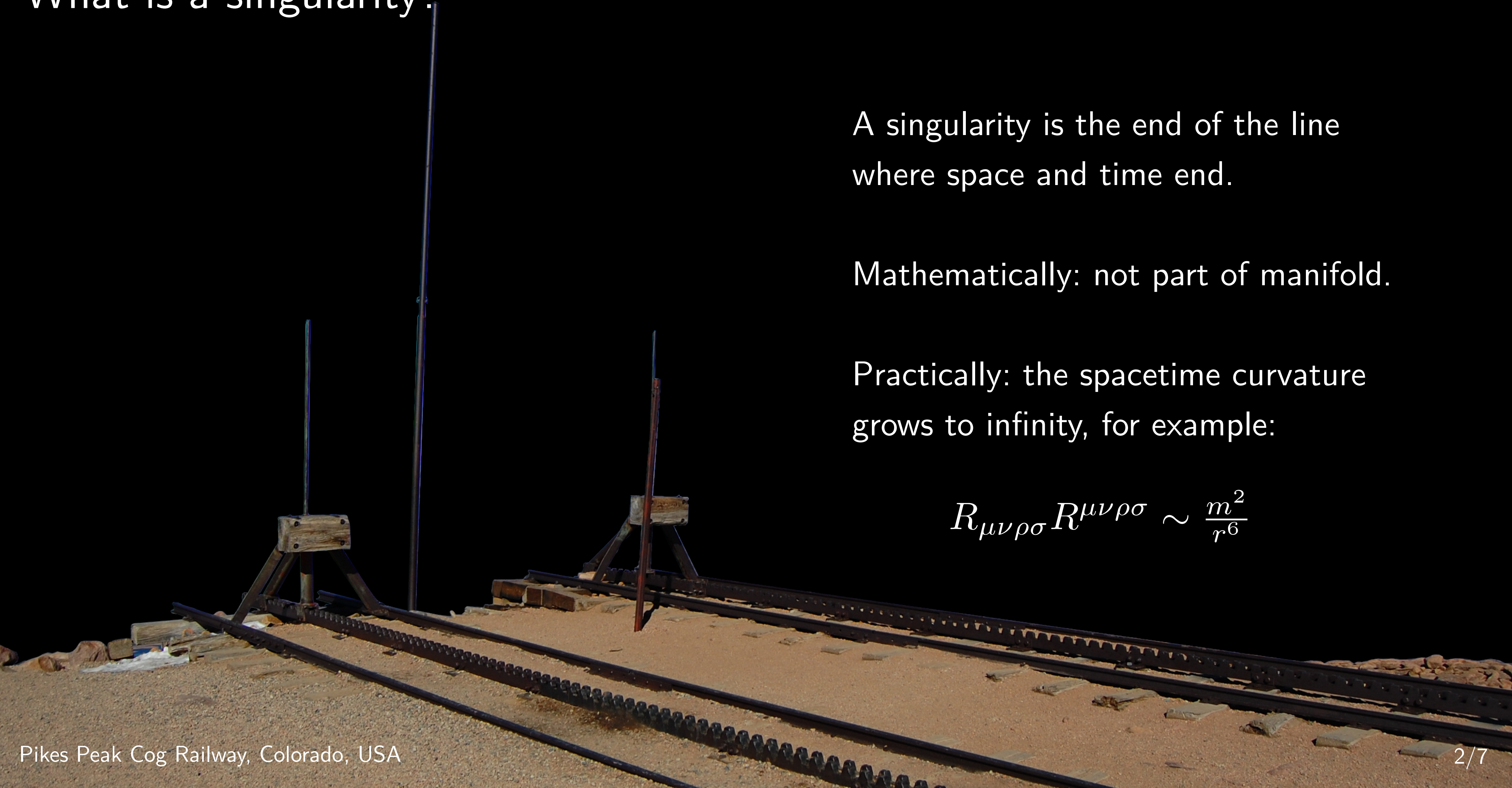

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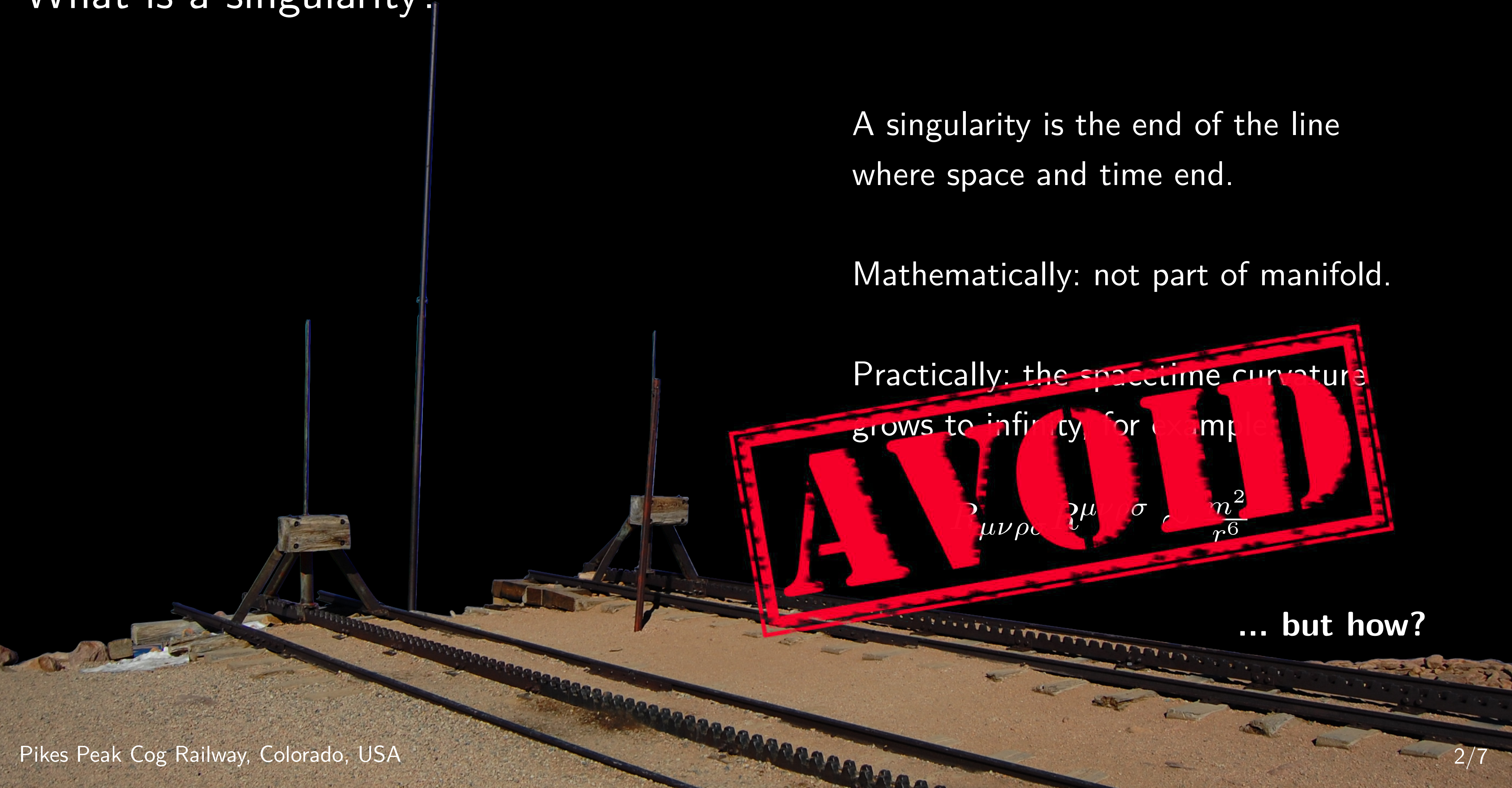

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... but how?

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A simple example: the gravitational potential of a point mass

The gravitational potential of a point-particle is singular at the origin:

$$\nabla^2 \phi = 4\pi G m \delta(\boldsymbol{r}) \quad \longrightarrow \quad \phi(r) = \frac{-Gm}{r}$$

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Modify the Poisson equation by an operator $\hat{F}$ that contains **infinitely many derivatives**:

$$\hat{F}\,\nabla^2 \phi = 4\pi G m \delta(\boldsymbol{r})\,, \qquad \hat{F} = \exp(\ell^2 \nabla^2) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\ell^2 \nabla^2\right)^n$$

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Okay, but why does that solve our problem?

Let's do a simple "back of the envelope" calculation!

Back of the envelope calculation: an effective delta source!

Let's consider a simple one-dimensional example:

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What have we done? The delta-shaped source has been smoothed!

The result is just a Gaussian bell curve.

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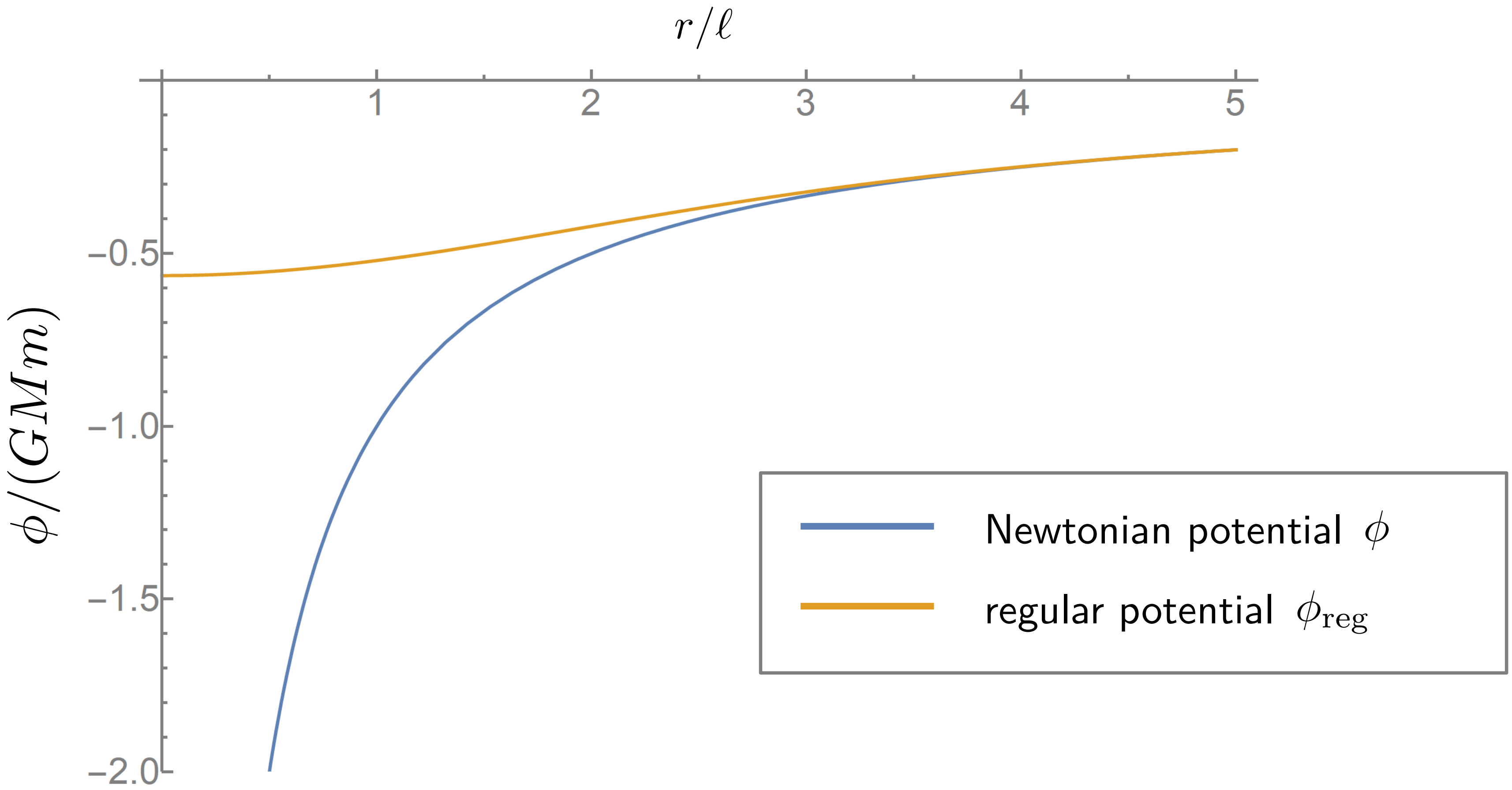

Belt grinder, Matthias Wandel (2018).

Application to black holes?

Infinite-derivative operators ("mathematical sandpaper") can be used to smooth sharp, local quantities over a characteristic length scale ℓ. This works well for linear equations. Not linear:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} \;=\; \frac{8\pi G}{c^3}T_{\mu\nu}$$

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They are ten non-linear partial differential equations for the ten functions $\mathrm{g}_{\mu\nu}(x)$, if one provides the energy-momentum (energy density, pressure, shear,...) $\mathrm{T}_{\mu\nu}(x)$.

Thank you for your attention!

Abstract

In my ten minutes I would like to present the idea of smoothing black hole singularities by using infinite-derivative operators. I will show in a simple calculation how these operators indeed smooth delta-shaped potentials and act as "mathematical sandpaper." Applying this to black holes is difficult, but I will try to outline the main conceptual steps on the journey towards singularity-free black holes.