Effects of Non-locality in Gravity and Quantum Theory

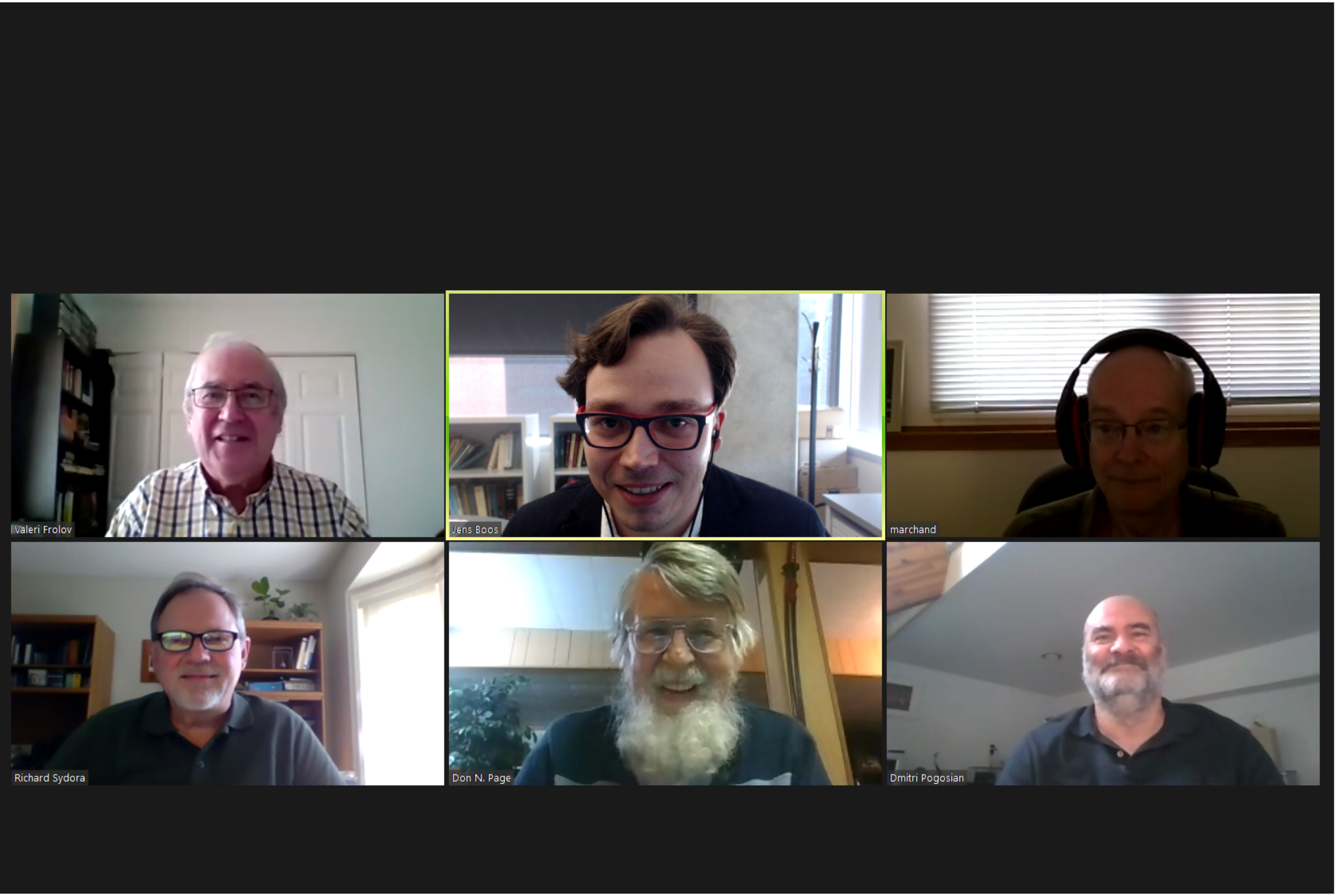
Jens Boos
jboos@wm.edu
High Energy Theory, William & Mary, VA, United States

Ph.D. Thesis, University of Alberta, supervisor: Valeri P. Frolov

I would like to thank Mark Walton and Andrew Frey for the kind invitation.

DTP Annual Business Meeting, Canadian Association of Physicists
2021 DTP/WITP P R Wallace PhD thesis prize talk
Monday, June 14, 2021, 1:20pm

Defending a PhD during the pandemic...



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in Gravity and Quantum Theory

A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of Doctor of Philosophy

University of Alberta
Department of Physics
Edmonton, Alberta
June 12, 2020

Author
Jens Boos

Supervisor
Prof. Valeri P. Frolov

Major open problems in gravitational physics

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There are several long-standing major open problems in gravitational physics:

- Black holes contain singularities (where spacetime curvature grows to infinity).
- Black holes evaporate, which may be in conflict with unitarity and locality.
- Our entire Universe is thought to evolve from a cosmological singularity ("big bang").

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It is conceivable that (some of) these problems can be solved by **new physics** that becomes relevant at very small distances and high energies, perhaps in an effective field theory of quantum gravity.

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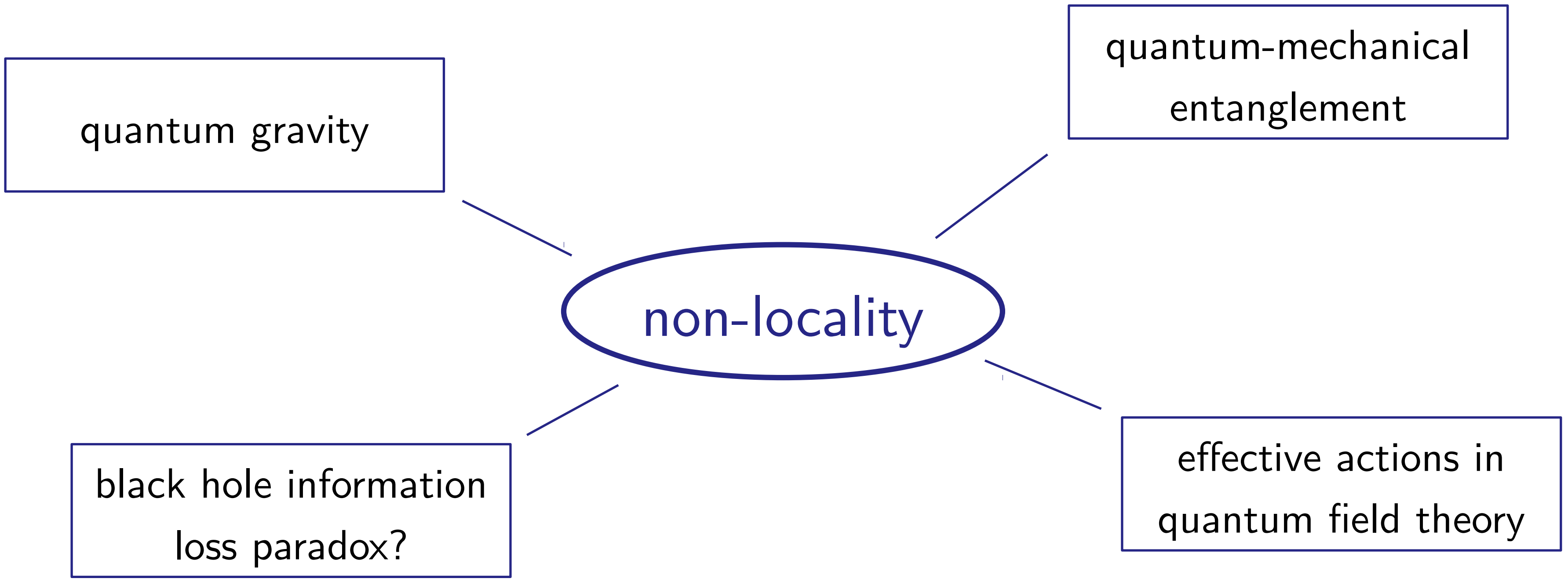
plan for today:

I want to show you that non-locality can remove the Newtonian $1/r$-singularity.

Non-locality touches many different areas of physics



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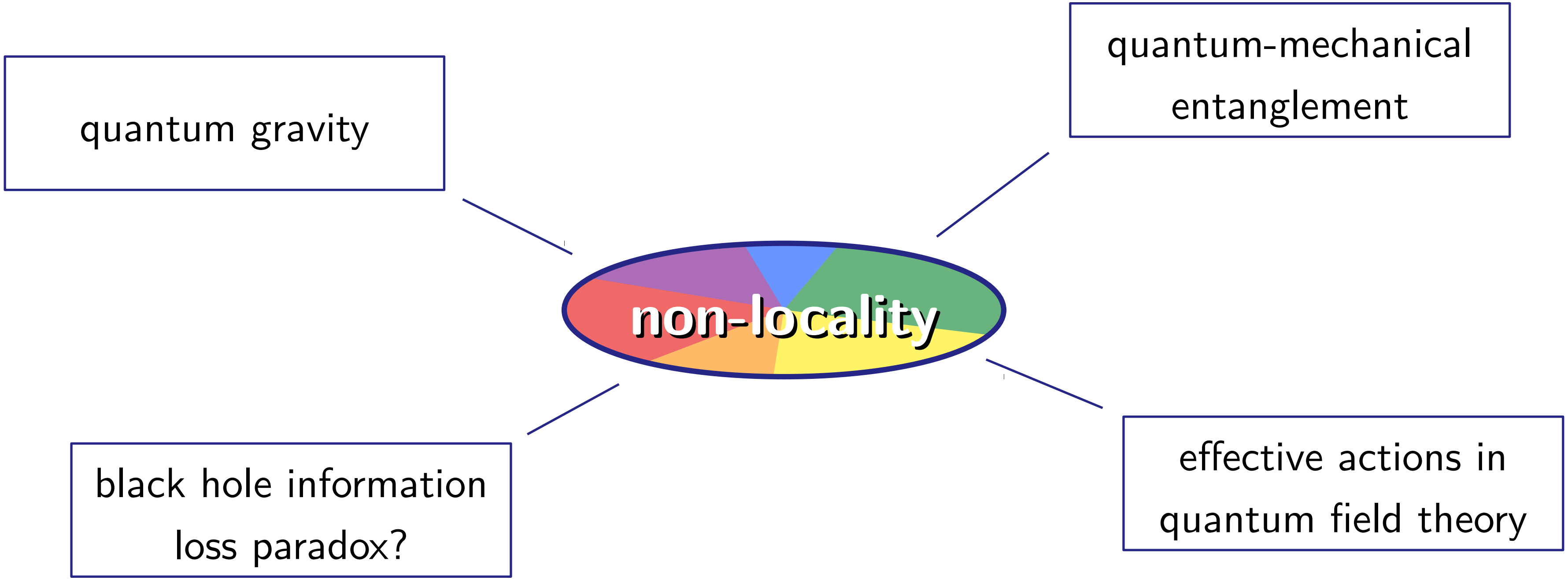

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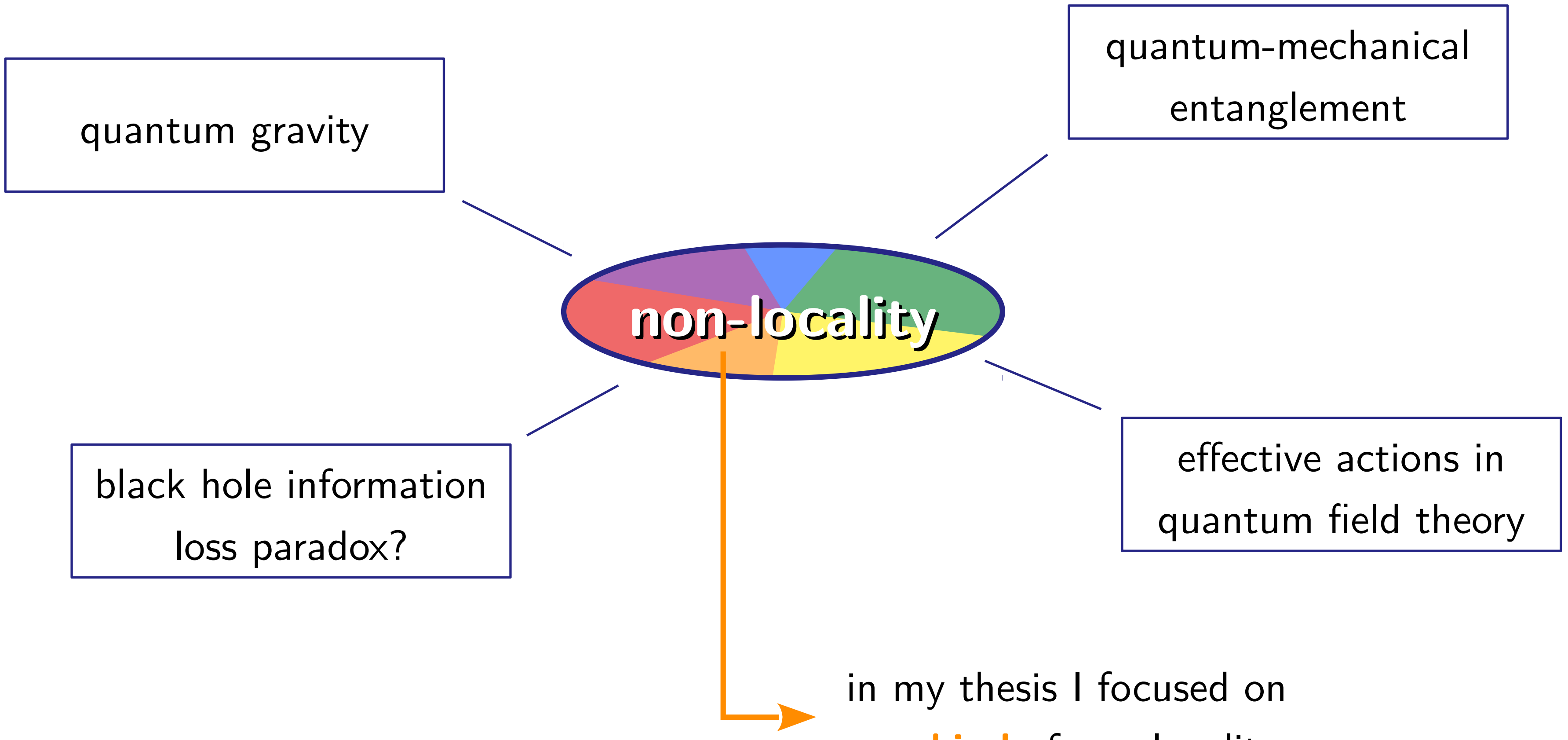

in my thesis I focused on **one kind** of non-locality

A somewhat minimal approach to non-locality

List of assumptions:

- defined in weak-field regime in Minkowski spacetime
- keep Lorentz invariance
- recover local theory at large distances/low energies
- no new degrees of freedom

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This solves the Newtonian **singularity problem**.

$$\boxed{\nabla^2\phi = 4\pi G\, M\, \delta^{(3)}(\boldsymbol{x}) \qquad \rightarrow \qquad \phi = -\frac{GM}{r}}$$

Non-locality in real space

A simple starting point is a scalar action:

$$S[\phi] = \int \mathrm{d}^4x \left[\frac{1}{2}\phi(x)(\Box - m^2)\phi(x) - V\big(\phi(x)\big) \right]$$

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Re-write this by inserting the **identity**:

$$S[\phi] = \int \mathrm{d}^4x \left\{\int \mathrm{d}^4y \left[\frac{1}{2}\phi(x)(\Box - m^2)\delta^{(4)}(x-y)\phi(y)\right] - V\big(\phi(x)\big)\right\}$$

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Can include non-locality by replacing the δ-function with a **non-local kernel**:

$$S[\phi] = \int \mathrm{d}^4x \left\{\int \mathrm{d}^4y \left[\frac{1}{2}\phi(x)(\Box - m^2)K(x-y)\phi(y)\right] - V\big(\phi(x)\big)\right\}$$

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Kinetic term in Fourier space:

$$(-1)(p^2+m^2)f(-p^2)$$

each zero = propagating mode

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Kinetic term in Fourier space:

$$(-1)(p^2+m^2)f(-p^2) \longrightarrow f(\Box) = e^{\ell^2\Box}$$

each zero = propagating mode

no additional particles!

("infinite derivatives")

Example: non-local Newtonian gravity

Local equations:

$$\boxed{\nabla^2\phi = 4\pi G\, M\,\delta^{(3)}(\boldsymbol{x}) \qquad \rightarrow \qquad \phi = -\frac{GM}{r}}$$

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$$\boxed{\nabla^2 e^{\ell^2 \nabla^2} \phi = 4\pi G\, M \delta^{(3)}(\boldsymbol{x}) \quad \to \quad \phi = -\frac{GM}{r} \,\mathrm{erf}\left(\frac{r}{2\ell}\right)}$$

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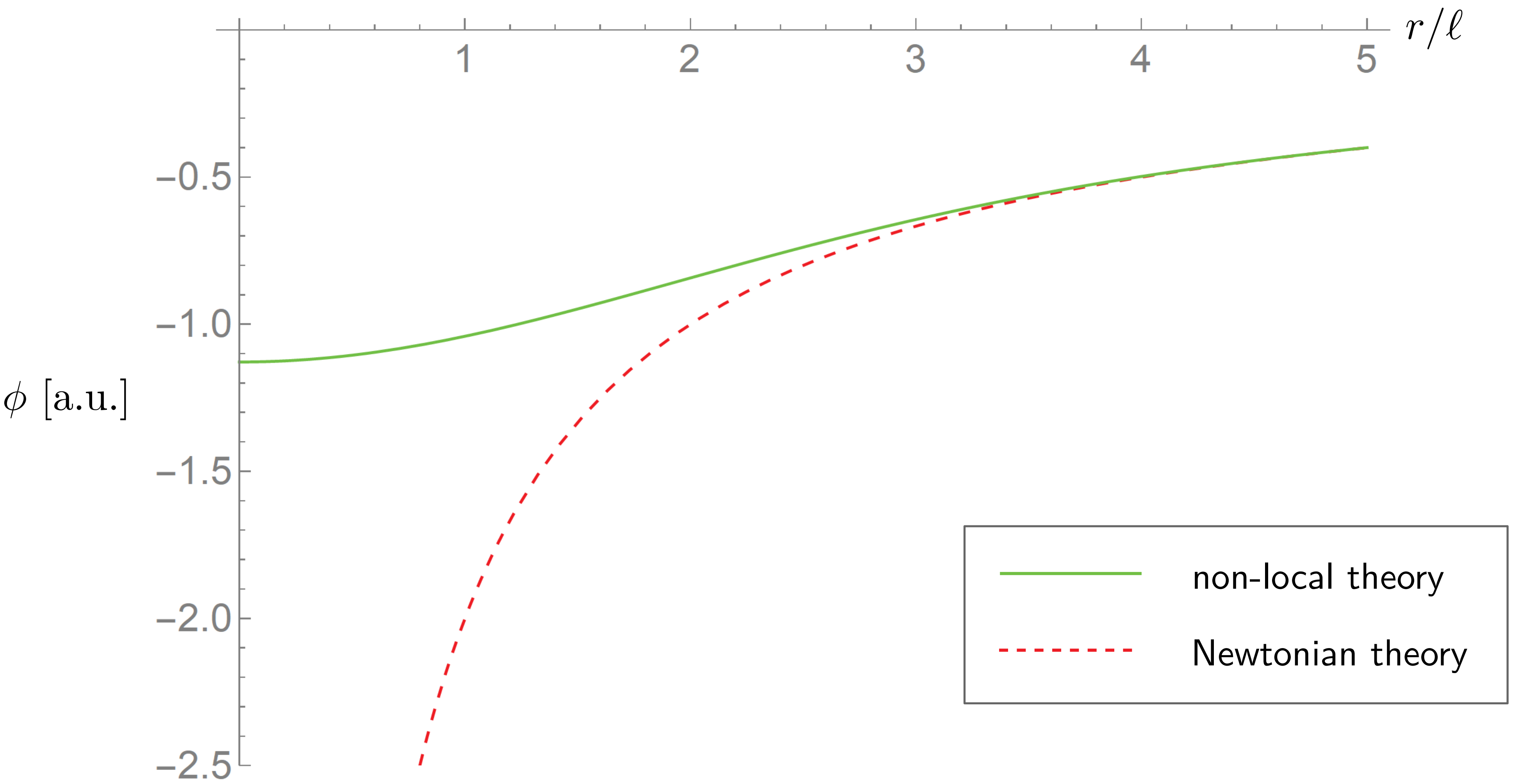
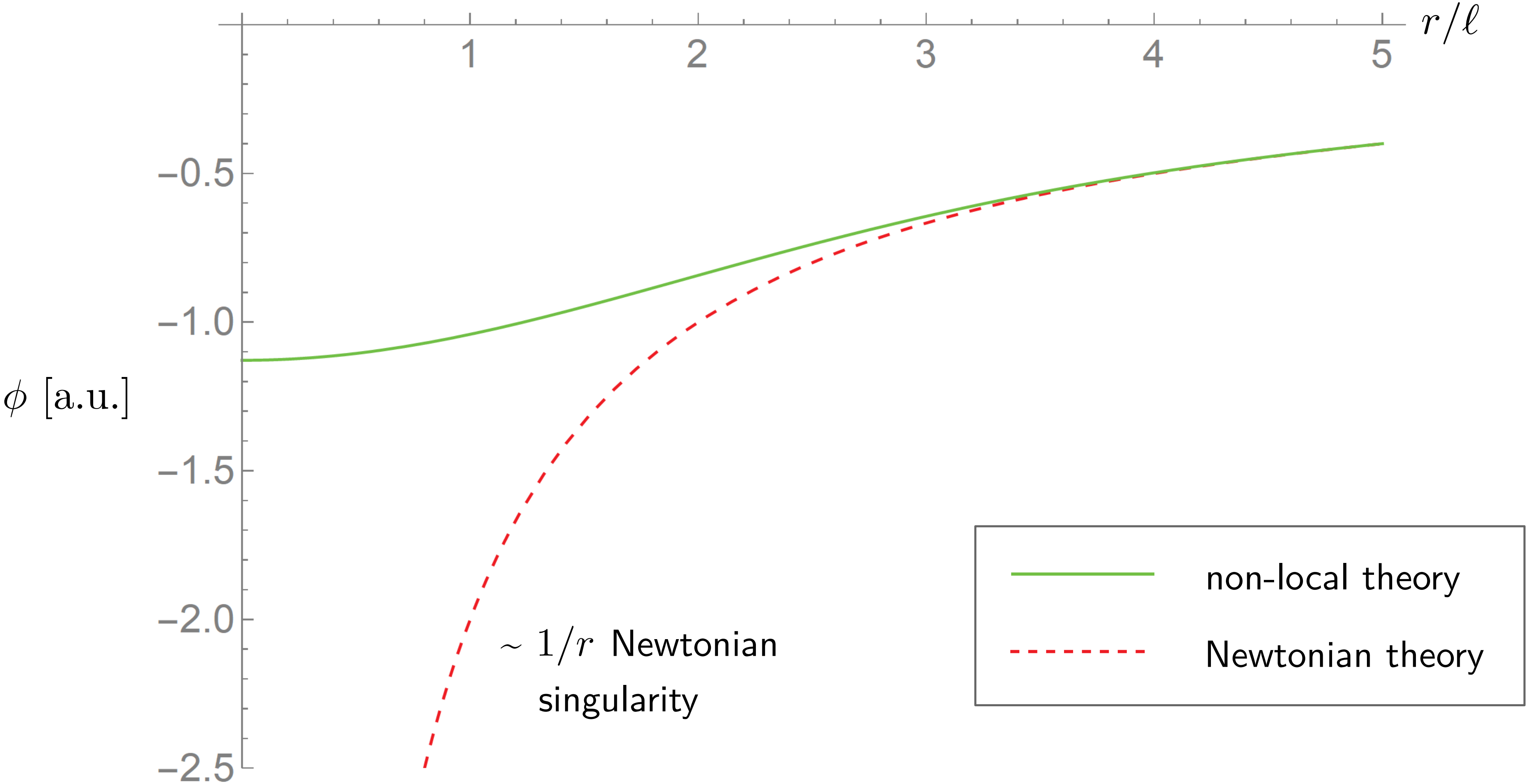
For this example, the integral kernel is a Gaussian with width of ℓ, the scale of non-locality.

$$f(\Box) = f(\nabla^2) = e^{\ell^2\nabla^2} \quad \leftrightarrow \quad K(x-y) = \frac{1}{(4\pi\ell^2)^{3/2}} e^{-\frac{|x-y|^2}{2\ell^2}}$$



r/ℓ
1
2
3
4
5
−0.5
−1.0
−1.5
−2.0
−2.5
ϕ [a.u.]
non-local theory
Newtonian theory



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$\sim 1/r$ Newtonian singularity
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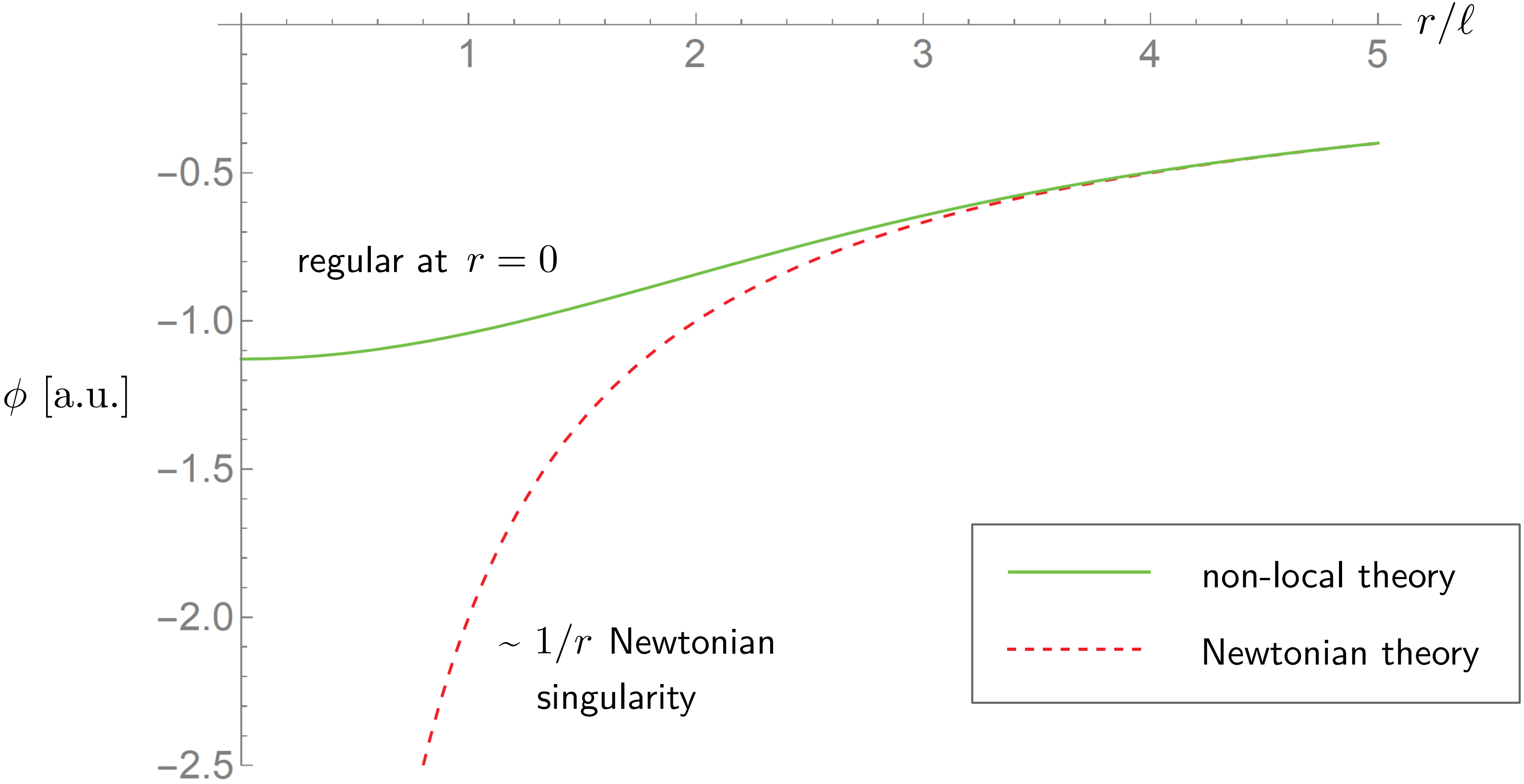

r/ℓ
1
2
3
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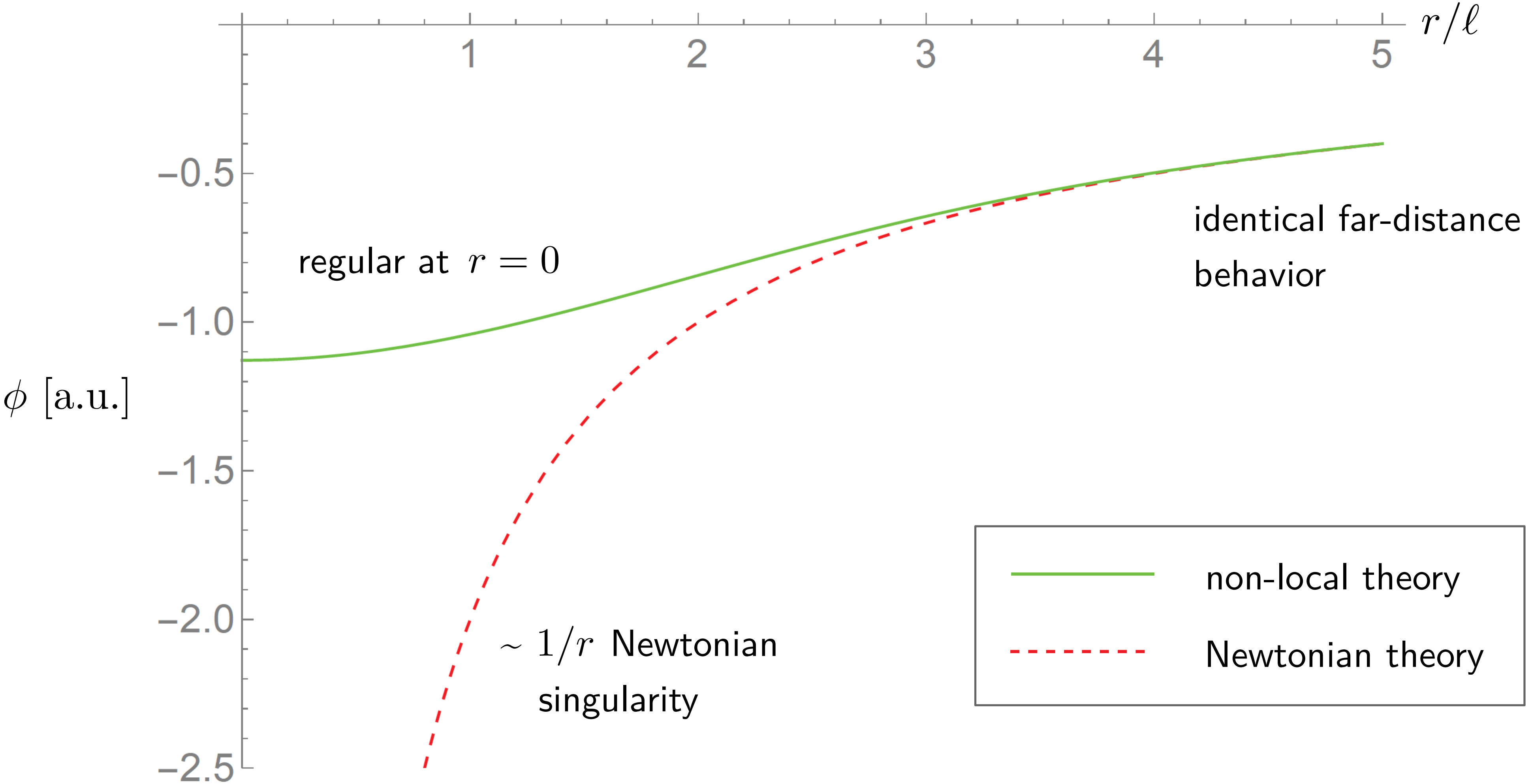
−1.5
−2.0
−2.5
ϕ [a.u.]
regular at $r = 0$
$\sim 1/r$ Newtonian singularity
non-local theory
Newtonian theory



r/ℓ
1
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3
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5
−0.5
−1.0
−1.5
−2.0
−2.5
ϕ [a.u.]
regular at $r = 0$
identical far-distance behavior
$\sim 1/r$ Newtonian singularity
non-local theory
Newtonian theory

Many other applications of this type of non-locality:

During my PhD, with Valeri P. Frolov, Andrei Zelnikov, Jose Pinedo Soto (all U of Alberta):

- regular particles and p-branes (1802.09573), regular rotating strings (2003.13847)
- spatial “Friedel” oscillations around point particles (1804.00225)
- new absorption lines in quantum scattering (1805.01875)
- regular vacuum fluctuations (1901.07096) and thermal fluctuations (1904.07917)
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- regular gyratons in gravity (2004.07420) and electrodynamics (2012.05347)
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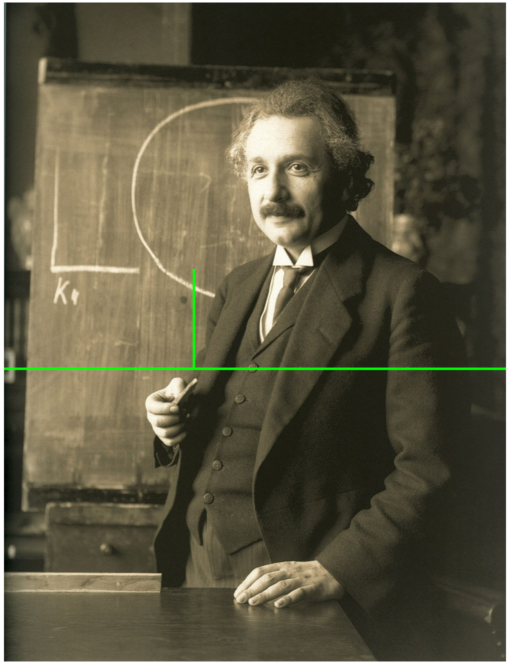
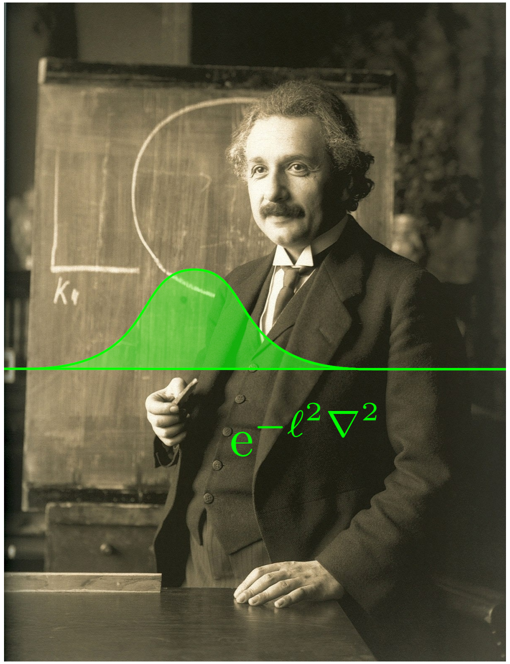
- gravito-electromagnetism (2103.10555) → PR Wallace did his PhD on electromagnetism in GR!
- hierarchy problem (2104.11195)

Main insights about non-locality:





- intuition from non-local Green functions
- non-locality regularizes potentials
- new physics $\neq$ new particles!

Thank you for again for the kind invitation and for your attention.