Asymptotic nonlocality

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Based on work with Christopher D. Carone.

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Based on...

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Asymptotic nonlocality in gauge theories

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...and a third one, “On asymptotic nonlocality in non-Abelian gauge theories” 2112.05270 [hep-ph].

Motivation

Physical observables should be finite, but infinities appear throughout theoretical physics:

- Newtonian potential, Coulomb potential
- black holes, cosmology
- electrostatic energy of a point charge
- self-energies in quantum field theory
- ...

Two paths: deal with them (renormalization) or reduce degree of divergence.

Second type: supersymmetry, but also Pauli-Villars regularization, higher-derivative theories, Lee-Wick theories, nonlocal theories.

Today I want to focus on nonlocality. Why?

Motivation

Nonlocality plays an important role in modern physics:

- Entanglement is a nonlocal phenomenon in quantum theory.
- Many effective actions in quantum field theory are nonlocal.
- Nonlocality may solve the black hole information loss problem.
- It is probably impossible to define local observables in quantum gravity.

But: nonlocality also challenges many of our "standard" notions in theoretical physics.

- Causality is typically violated at some scale (and perhaps beyond).
- Variational principle is not necessarily self-consistent.
- Notion of a "local particle" is difficult to define.

Today: let us consider a class of theories that has a **nonlocal limit point**.

Nonlocal models

Consider models that are described by an infinite-derivative modification inside the kinetic term:

$$\mathcal{L} = -\frac{1}{2}\phi\Box f(\Box)\phi - V(\phi)\,,$$

The object $f(\Box)$ is called a form factor and mediates deviations from local theories.
The property $f \neq 0$ guarantees the absence of new perturbative degrees of freedom.

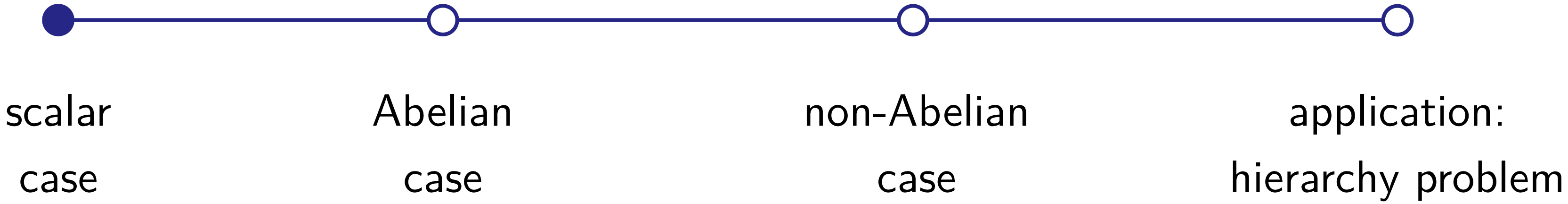
Main features:

- Nonlocality regularizes point-like potentials in classical theories.
- Euclidean UV behavior of propagators is drastically improved.
- IR behavior is identical to local theories.

Open problems:

- The role of Wick rotation remains somewhat unclear.
- Usually the form factor has to be chosen by hand.

Asymptotic nonlocality



Scalar theory: Lagrangian and basic properties

Non-generic Lagrangian for N fields ϕ_j and $N-1$ auxiliary fields χ_j :

$$\mathcal{L} = -\frac{1}{2}\phi_1 \Box \phi_N - V(\phi_1) - \sum_{j=1}^{N-1} \chi_j \left(\Box \phi_j - \frac{\phi_{j+1} - \phi_j}{a_j^2} \right)$$

The auxiliary χ-fields can be integrated out exactly. Result:

$$\phi_{j+1} = (1 + a_j^2 \Box)\phi_j$$

Repeated application:

$$\phi_N = \left[\prod_{j=1}^{N-1} (1 + a_j^2 \Box) \right] \phi_1$$

Scalar theory: higher derivatives and nonlocal limit point

Plug the previous into the original Lagrangian:

$$\mathcal{L} = -\frac{1}{2}\phi_1 \Box \left[\prod_{j=1}^{N-1} (1 + a_j^2 \Box) \right] \phi_1 - V(\phi_1)$$

This is a typical higher-derivative Lagrangian. Now define $\ell_j^2 \equiv a_j^2 (N-1)$ such that

$$\prod_{j=1}^{N-1} (1 + a_j^2 \Box) = \prod_{j=1}^{N-1} \left(1 + \frac{\ell_j^2}{N-1} \Box \right) \approx \left(1 + \frac{\ell^2}{N-1} \Box \right)^{N-1} = \exp(\ell^2 \Box)\,.$$

Assume $N \to \infty$ and $\ell_j \to \ell$ in this limit.

This is the nonlocal limit point.

Scalar theory: higher derivatives and nonlocal limit point

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$$\mathcal{L} = -\frac{1}{2}\phi_1 \Box \left[\prod_{j=1}^{N-1} (1 + a_j^2 \Box) \right] \phi_1 - V(\phi_1)$$

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Assume $N \to \infty$ and $\ell_j \to \ell$ in this limit.

This is the nonlocal limit point.

ℓ is an emergent scale that is not present in the original Lagrangian.

Scalar theory: Lee-Wick picture

Consider the quadratic part of the initial Lagrangian,

$$\mathcal{L}_{\text{quad}} = -\frac{1}{2}\phi_1\Box\phi_N - \sum_{j=1}^{N-1}\chi_j\left(\Box\phi_j - \frac{\phi_{j+1}-\phi_j}{a_j^2}\right) = -\frac{1}{2}\Psi^T(K\Box + M)\Psi\,,$$

with the collection of fields $\Psi = (\phi_1, \chi_1, \psi_2, \chi_2, \dots, \phi_{N-1}, \chi_{N-1}, \phi_N)$. Block-diagonal form:

$$K = \begin{pmatrix} 1 & & & & 0 \\ & (-1)^1 & & & 0 \\ & & \ddots & & \vdots \\ & & & (-1)^{N-1} & 0 \\ 0 & 0 & \dots & 0 & X \end{pmatrix}\,, \quad M = \begin{pmatrix} 0 & & & & 0 \\ & (-1)^1 m_1^2 & & & 0 \\ & & \ddots & & \vdots \\ & & & (-1)^{N-1} m_{N-1}^2 & 0 \\ 0 & 0 & \dots & 0 & Y \end{pmatrix}\,,$$

Sectors: one physical field, N-1 Lee-Wick partners, N-1 auxiliary fields can be integrated out.

Schematic overview

$$\mathcal{L}_{\text{quad}} = -\frac{1}{2}\,\phi_1\Box\phi_N - \sum_{j=1}^{N-1}\chi_j\,\left[\Box\phi_j - (\phi_{j+1}-\phi_j)/a_j^2\right]$$

fields: $\phi_1,\,\chi_1,\,\ldots,\,\phi_{N-1},\,\chi_{N-1},\,\phi_N$

integate out all χ_j

field redefinition $\Psi = (\Psi_1,\,\ldots,\,\Psi_{2N-1})$

$$\mathcal{L}_{\text{quad}} = -\frac{1}{2}\,\phi_1\Box\prod_{j=1}^{N-1}\left(1+a_j^2\Box\right)\phi_1$$

fields: ϕ_1

$$\mathcal{L}_{\text{quad}} = -\frac{1}{2}\sum_{i,j=1}^{2N-1}\Psi^i(K_i{}^j\Box + M_i{}^j)\Psi_j$$

fields: $\Psi_1,\,\ldots,\,\Psi_{2N-1}$

$N\to\infty\,,$

$a_j^2 \to \dfrac{\ell^2}{N-1}$

integrate out $\Psi_{N+1},\,\ldots,\,\Psi_{2N-1}$

$$\mathcal{L}_{\text{quad}} = -\frac{1}{2}\,\phi_1\Box e^{\ell^2\Box}\phi_1$$

fields: ϕ_1

$$\ell^2 \approx \frac{N}{m_1^2}$$

$$\mathcal{L}_{\text{quad}} = -\frac{1}{2}\sum_{j=1}^{N}(-1)^{j+1}\Psi_j(\Box+m_j^2)\Psi_j\,,$$

fields: $\Psi_1,\,\ldots,\,\Psi_N$

Asymptotic nonlocality

scalar case	Abelian case	non-Abelian case	application: hierarchy problem

Abelian gauge theory: Lagrangian and basic properties

Recall the initial Lagrangian from scalar field theory:

$$\mathcal{L} = -\frac{1}{2}\phi_1 \Box \phi_N - V(\phi_1) - \sum_{j=1}^{N-1} \chi_j \left(\Box \phi_j - \frac{\phi_{j+1} - \phi_j}{a_j^2} \right)$$

Simple extension to Abelian gauge theory:

$$\mathcal{L}_{\text{gauge}} = \frac{1}{2} \hat{A}^1_\mu \mathcal{O}^\mu_\nu \hat{A}^\nu_N + \sum_{j=1}^{N-1} \chi^j_\mu \left(O^\mu_\nu \hat{A}^\nu_j - \frac{\hat{A}^\mu_{j+1} - \hat{A}^\mu_j}{a_j^2} \right)$$

Here, $\mathrm{O}^\mu_\nu = \delta^\mu_\nu \Box - \partial^\mu \partial_\nu$ is the differential operator, and the covariant derivative is defined via the vector potential $\hat{A}^1_\mu$. Gauge trafo: $\hat{A}^j_\mu \to \hat{A}^j_\mu + \partial_\mu \lambda\,, \quad \chi^j_\mu \to \chi^j_\mu\,.$

Abelian gauge theory: Lagrangian and basic properties

Simple extension to Abelian gauge theory:

$$\mathcal{L}_{\text{gauge}} = \frac{1}{2}\hat{A}^1_\mu \mathcal{O}^\mu_\nu \hat{A}^\nu_N + \sum_{j=1}^{N-1} \chi^j_\mu \left(O^\mu_\nu \hat{A}^\nu_j - \frac{\hat{A}^\mu_{j+1} - \hat{A}^\mu_j}{a_j^2} \right)$$

The auxiliary χ-fields can be integrated out exactly. Result:

$$\hat{A}^{j+1}_\mu = (\delta^\nu_\mu + a_j^2 \mathcal{O}^\nu_\mu)\hat{A}^j_\nu$$

Repeated application again gives

$$\hat{A}^N_\mu = \left[\prod_{j=1}^{N-1} (\delta^\nu_\mu + a_j^2 \mathcal{O}^\nu_\mu) \right] \hat{A}^1_\nu \,.$$

Abelian gauge theory: higher derivatives and nonlocal limit point

Plug the previous into the original Lagrangian and use $(\mathcal{O}^\mu_\nu)^N = \Box^{N-1}\mathcal{O}^\mu_\nu$:

$$\mathcal{L}_{\text{gauge}} = \frac{1}{2}\hat{A}^1_\mu \mathcal{O}^\mu_\nu \left[\prod_{j=1}^{N-1}(1+a_j^2\Box)\right]\hat{A}^\nu_1 = -\frac{1}{4}\hat{F}^1_{\mu\nu}\left[\prod_{j=1}^{N-1}(1+a_j^2\Box)\right]\hat{F}^{\mu\nu}_1$$

This is a typical higher-derivative Lagrangian. Now define $\ell_j^2 \equiv a_j^2(N-1)$ such that

$$\prod_{j=1}^{N-1}(1+a_j^2\Box) = \prod_{j=1}^{N-1}\left(1+\frac{\ell_j^2}{N-1}\Box\right) \approx \left(1+\frac{\ell^2}{N-1}\Box\right)^{N-1} = \exp(\ell^2\Box)\,.$$

Assume $N\to\infty$ and $\ell_j\to\ell$ in this limit.

This is the nonlocal limit point.

Abelian gauge theory: Lee-Wick picture

Consider the quadratic part of the initial Lagrangian,

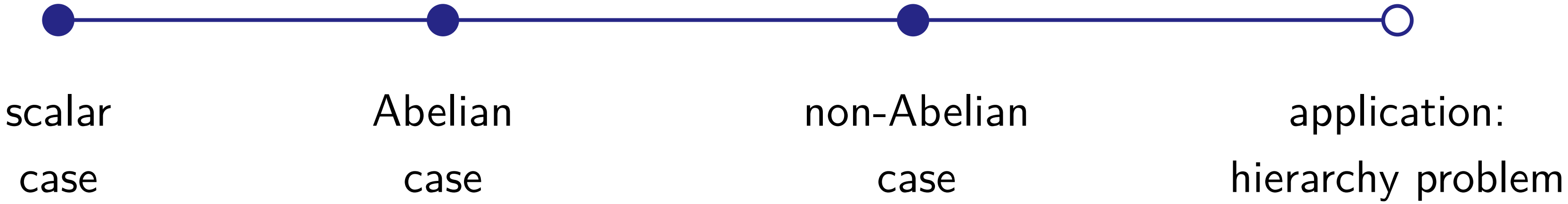
$$\mathcal{L}_{\text{quad}} = -\frac{1}{2}\hat{A}^1_\mu \mathcal{O}^\mu_\nu \hat{A}^\nu_N - \sum_{j=1}^{N-1} \chi^j_\mu \left(O^\mu_\nu \hat{A}^\nu_j - \frac{\hat{A}^\mu_{j+1} - \hat{A}^\mu_j}{a_j^2} \right) = \frac{1}{2}\underline{\hat{A}}^T_\mu (K\mathcal{O}^\mu_\nu + M\delta^\mu_\nu)\underline{\hat{A}}^\nu \,,$$

with the collection of fields $\underline{\hat{A}}_\mu = (\hat{A}^1_\mu, \chi^1_\mu, \hat{A}^2_\mu, \chi^2_\mu, \ldots, \hat{A}^{N-1}_\mu, \chi^{N-1}_\mu, \hat{A}^N_\mu)$. Block-diagonal form:

$$K = \begin{pmatrix} 1 & & & & 0 \\ & (-1)^1 & & & 0 \\ & & \ddots & & \vdots \\ & & & (-1)^{N-1} & 0 \\ 0 & 0 & \ldots & 0 & X \end{pmatrix}, \quad M = \begin{pmatrix} 0 & & & & 0 \\ & (-1)^1 m_1^2 & & & 0 \\ & & \ddots & & \vdots \\ & & & (-1)^{N-1} m_{N-1}^2 & 0 \\ 0 & 0 & \ldots & 0 & Y \end{pmatrix},$$

Sectors: one physical field, N-1 Lee-Wick partners, N-1 auxiliary fields can be integrated out.

Asymptotic nonlocality



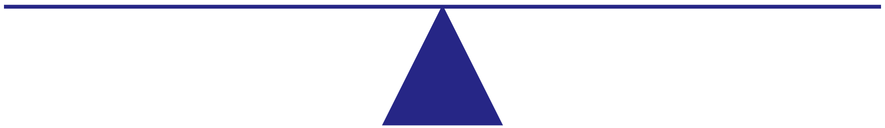
Non-Abelian gauge theory: what is the same, what is different?

Complication: purely quadratic term is not gauge-invariant due to gluon self-interaction.
Formulation of an auxiliary theory with χ-fields appears to be non-trivial.

Our perspective: look at higher-derivative picture using "almost quadratic" $\mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu}$ term.
Alternative: try to generate an equivalent Lee-Wick Lagrangian with massive vectors.

simple higher-derivative kinetic term

simple Lee-Wick self-interactions



For explicit examples see Grinstein-O'Connell-Wise PRD 2008 (N=2),
Carone-Lebed PRD 2009 (N=3).

Non-Abelian gauge theory: higher derivatives and nonlocal limit point

Start with the following Lagrangian:

$$\mathcal{L}_{\mathrm{gauge}} = -\frac{1}{2}\mathrm{Tr}\, F_{\mu\nu}\left[\prod_{j=1}^{N-1}(1+a_j^2\underline{\Box})\right]F^{\mu\nu}\,, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]\,.$$

Here, $\underline{\Box}$ is the gauge-covariant d'Alembertian. Non-local limit point is obtained as shown before. Coupling to asymptotically nonlocal complex scalar via standard covariant derivative,

$$\mathcal{L}_{\mathrm{matter}} = -\phi^*(\underline{\Box} + m_\phi^2)\left[\prod_{j=1}^{N-1}(1+a_j^2\underline{\Box})\right]\phi - V(\phi)\,.$$

Read off Feynman rules; new scalar-gluon vertex functions due to presence of higher derivatives.

Asymptotic nonlocality

scalar case

Abelian case

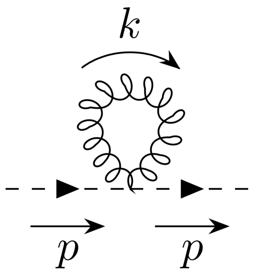
non-Abelian case

application: hierarchy problem

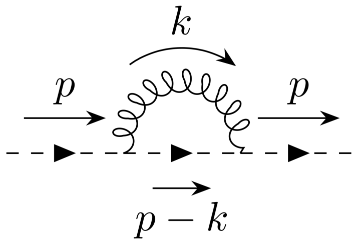
Application: electroweak hierarchy problem

Can compute the on-shell scalar self-energy analytically at 1-loop. For $N \to \infty$ one finds

$$-iM^2(p^2 = m_\phi^2) =$$



$$+$$



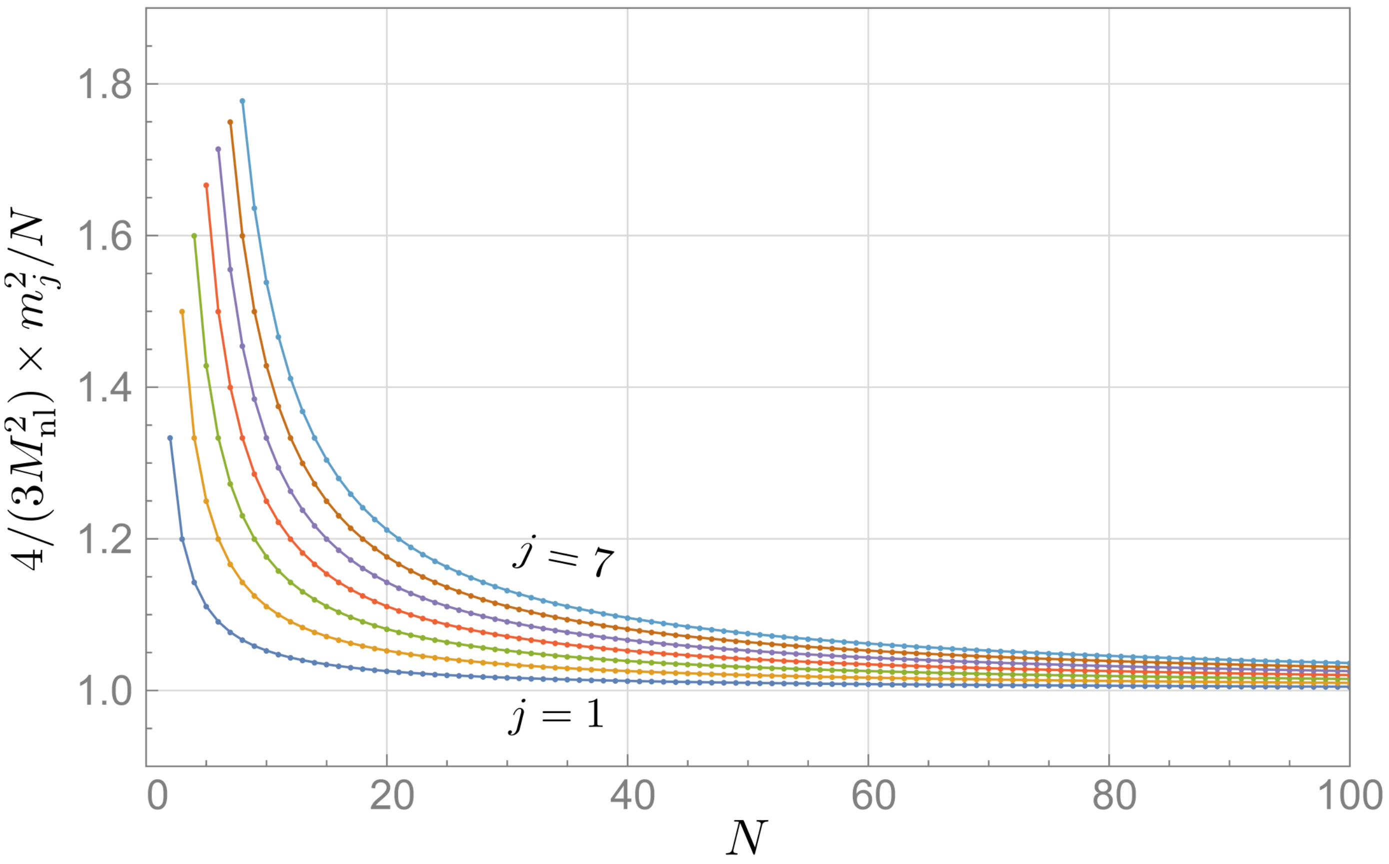
$$= -i\frac{3g^2}{16\pi^2}\frac{1}{\ell^2}C_2 e^{-\ell^2 m_\phi^2}\left[1 + F(m_\phi^2 \ell^2)\right] \, ,$$

$$F(z) = \frac{1}{6}\left[(z^2 - 2z)e^z \mathrm{Ei}_1(z) - z\right] + \frac{\sqrt{z}}{6}\left[G^{22}_{23}\left(z \,\middle|\, \begin{matrix} -\frac{1}{2}, 1; \\ \frac{3}{2}, \frac{1}{2}; 0 \end{matrix}\right) + 4\, G^{22}_{23}\left(z \,\middle|\, \begin{matrix} -\frac{1}{2}, 1; \\ \frac{1}{2}, \frac{1}{2}; 0 \end{matrix}\right)\right] \, .$$

Main results: Scale is set by emergent scale ℓ.

This scale is **hierarchically lower** than lightest Lee-Wick particle.

Application: electroweak hierarchy problem



The mass parametrization $m_j^2 = 1/a_j^2 = 3N/[2\ell^2(2 - j/N)]$ is exactly solvable.

Application: electroweak hierarchy problem

Some technical details on higher-derivative propagators and partial fraction decompositions:

$$D^{ab}_{\mu\nu}(p) = -i\frac{\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\left[1 - \xi f(-p^2)\right]}{p^2 f(-p^2)}\delta^{ab}\,, \qquad \frac{1}{f(-p^2)} = \sum_{j=1}^{N-1}\frac{b_j}{p^2 - m_j^2}\,.$$

Cancellation rules:

$$\sum_{j=1}^{N-1} b_j m_j^{2n} = 0 \quad \text{for } n = 0, \ldots, N-2\,.$$

Sequence of computations:

partial fraction → Wick rotate → dim-reg → cancellation rules

Pole prescriptions? Can use Lee-Wick prescription for on-shell self-energy, but no pinching on-shell.

Asymptotic nonlocality

scalar case

Abelian case

non-Abelian case

application: hierarchy problem

Discussion of results

We have constructed a class of theories with a nonlocal limit point.
In the scalar and Abelian case it is possible to alternatively work in a Lee-Wick picture.

Asymptotic nonlocality: appearance of a nonlocal scale that regulates loop diagrams.

Theories are also interesting away from the limit point, due to central relation $M_{\mathrm{LW}} \sim N M_{\mathrm{nl}}$.
New physics can be "mediated" by massive unobservable particles, albeit at the cost of locality.

Future work: signatures at collider physics? Role of form factors?

Bedankt & thank you very much!

Abstract

Asymptotic nonlocality

The role of nonlocality is often modeled via form factors that contain an infinite number of derivatives. Here, we consider a local theory with N coupled fields. Upon integrating out various degrees of freedom one obtains either (i) a Lee-Wick theory with resonances M_j, or (ii) a higher-derivative theory. There is now the opportunity to consider the following limit: as the number of particles N as well as the Lee--Wick mass resonances M_j tend to infinity, what if M_j/N remains finite? This emergent scale can be hierarchically lower than the Lee-Wick scale, and we show that it indeed plays the role of an emergent scale of nonlocality ("asymptotic nonlocality") in the large-N limit. We explore various applications of such asymptotic nonlocality in scalar field theory, Abelian gauge theory, and non-Abelian gauge theory, and comment on possible implications for the resolution of the electroweak hierarchy problem.

Based on work with Christopher D. Carone.