Classical aspects of Poincaré gauge theory of gravity

Jens Boos
jboos@perimeterinstitute.ca
Perimeter Institute for Theoretical Physics

Wednesday, Nov 11, 2015

Quantum Gravity group meeting
Perimeter Institute for Theoretical Physics

Outline

- Motivation
- Gauging the Poincaré group
- Riemann–Cartan geometry
- Example: Einstein–Cartan theory
- Deformed Lie algebra

Based on F. W. Hehl, P. von der Heyde, G. D. Kerlick and J. M. Nester,
"General Relativity with Spin and Torsion: Foundations and Prospects,"
Rev. Mod. Phys. **48** (1976) 393.

Motivation

History:

- field equations of General Relativity: Einstein (1915)
- discovery of spin: Uhlenbeck & Goudsmit (1925)
- description of spin: Pauli (1927), relativistic: Dirac (1928)
- gauge theories: Weyl (1918, 1929, 1950), Yang–Mills (1954)
- gravity as gauge theory: Utiyama (1956), Sciama (1960), Kibble (1961)

Motivation

History:
- field equations of General Relativity: Einstein (1915)
- discovery of spin: Uhlenbeck & Goudsmit (1925)
- description of spin: Pauli (1927), relativistic: Dirac (1928)
- gauge theories: Weyl (1918, 1929, 1950), Yang–Mills (1954)
- gravity as gauge theory: Utiyama (1956), Sciama (1960), Kibble (1961)

"Newton successfully wrote apple $=$ moon, but you cannot write apple $=$ neutron."

– J. L. Synge

Motivation

History:
- field equations of General Relativity: Einstein (1915)
- discovery of spin: Uhlenbeck & Goudsmit (1925)
- description of spin: Pauli (1927), relativistic: Dirac (1928)
- gauge theories: Weyl (1918, 1929, 1950), Yang–Mills (1954)
- gravity as gauge theory: Utiyama (1956), Sciama (1960), Kibble (1961)

"Newton successfully wrote apple $=$ moon, but you cannot write apple $=$ neutron."

– J. L. Synge

The Dirac equation, minimally coupled to gravity (Weyl 1929):

$$\mathrm{i}\gamma^{\alpha} e^{j}{}_{\alpha} \left(\partial_j + \tfrac{\mathrm{i}}{4}\Gamma_j\right)\Psi + m\Psi = 0$$

Problem: the tetrad has to be put into General Relativity by hand.

Motivation

History:
- field equations of General Relativity: Einstein (1915)
- discovery of spin: Uhlenbeck & Goudsmit (1925)
- description of spin: Pauli (1927), relativistic: Dirac (1928)
- gauge theories: Weyl (1918, 1929, 1950), Yang–Mills (1954)
- gravity as gauge theory: Utiyama (1956), Sciama (1960), Kibble (1961)

"Newton successfully wrote apple = moon, but you cannot write apple = neutron."

– J. L. Synge

The Dirac equation, minimally coupled to gravity (Weyl 1929):

$$i\gamma^\alpha e^j{}_\alpha \left(\partial_j + \tfrac{i}{4}\Gamma_j\right)\Psi + m\Psi = 0$$

Problem: the tetrad has to be put into General Relativity by hand.

What if spin had been discovered before General Relativity?

Would Einstein have applied the equivalence principle to a neutron instead?

What is the underlying gauge structure of gravity? (1/2)

Electrodynamics
- is invariant under the transformation $A_i \to A_i + \partial_i \chi$
- couples to a conserved current j_i

$\rightarrow$ description of U(1) gauge theory (cf. Weyl's erroneous attempt in 1918)

What is the underlying gauge structure of gravity? (1/2)

Electrodynamics
- is invariant under the transformation $A_i \to A_i + \partial_i \chi$
- couples to a conserved current j_i

$\rightarrow$ description of U(1) gauge theory (cf. Weyl's erroneous attempt in 1918)

A symmetry of a matter field (here: complex scalar field, spinor) gave rise to a force.

What is the underlying gauge structure of gravity? (1/2)

Electrodynamics
- is invariant under the transformation $\mathsf{A}_\mathsf{i} \to \mathsf{A}_\mathsf{i} + \partial_\mathsf{i}\chi$
- couples to a conserved current j_i

$\to$ description of U(1) gauge theory (cf. Weyl's erroneous attempt in 1918)

A symmetry of a matter field (here: complex scalar field, spinor) gave rise to a force.

General Relativity
- is invariant under diffeomorphisms $x^i \to x'^i \approx x^i + \epsilon^i$
- couples to a covariantly conserved energy-momentum $\Theta_{ij} = \Theta_{ji}$

$\to$ which gauge transformations on a matter field give rise to gravity?

What is the underlying gauge structure of gravity? (1/2)

Electrodynamics
- is invariant under the transformation $A_i \to A_i + \partial_i \chi$
- couples to a conserved current j_i

$\rightarrow$ description of U(1) gauge theory (cf. Weyl's erroneous attempt in 1918)

A symmetry of a matter field (here: complex scalar field, spinor) gave rise to a force.

General Relativity
- is invariant under diffeomorphisms $x^i \to x'^i \approx x^i + \epsilon^i$
- couples to a covariantly conserved energy-momentum $\Theta_{ij} = \Theta_{ji}$

$\rightarrow$ which gauge transformations on a matter field give rise to gravity?

Several possible answers: diffeomorphisms, translations, Lorentz rotations, Poincaré transformations, affine transformations, supersymmetric extensions, ...

What is the underlying gauge structure of gravity? (2/2)

Several possible answers:

- diffeomorphisms
- translations
- Lorentz rotations
- Poincaré transformations
- affine transformations
- supersymmetric extensions
- ...

What is the underlying gauge structure of gravity? (2/2)

Several possible answers:

- diffeomorphisms
- translations
- Lorentz rotations
- **Poincaré transformations**
- affine transformations
- supersymmetric extensions
- ...

What is the underlying gauge structure of gravity? (2/2)

Several possible answers:

- diffeomorphisms
- translations
- Lorentz rotations
- **Poincaré transformations**
- affine transformations
- supersymmetric extensions
- ...

Why the Poincaré group?

- is the symmetry group of Minkowski space
- knows about energy-momentum and orbital angular momentum
- allows coupling to spin angular momentum (microphysics, cf. Synge)
- mass-spin classification (Wigner 1939)

The Poincaré group

Poincaré group $= \{$n translations$\} \rtimes \{\frac{n(n-1)}{2}$ Lorentz transformations$\}$

Lie algebra: $[M_{\mu\nu}, M_{\alpha\beta}] = g_{\alpha[\mu}M_{\nu]\beta} - g_{\beta[\mu}M_{\nu]\alpha},$ semidirect product

$[M_{\mu\nu}, P_{\alpha}] = g_{\alpha[\mu}P_{\nu]},$

$[P_{\mu}, P_{\alpha}] = 0.$

A suitable real-space representation: $M_{ij} \sim x_{[i}\partial_{j]}$, $P_a \sim \partial_a$

The Poincaré group

Poincaré group $= \{\text{n translations}\} \rtimes \{\frac{n(n-1)}{2}$ Lorentz transformations$\}$

Lie algebra: $[M_{\mu\nu}, M_{\alpha\beta}] = g_{\alpha[\mu}M_{\nu]\beta} - g_{\beta[\mu}M_{\nu]\alpha}$, semidirect product

$$[M_{\mu\nu}, P_\alpha] = g_{\alpha[\mu}P_{\nu]},$$

$$[P_\mu, P_\alpha] = 0.$$

A suitable real-space representation: $M_{ij} \sim x_{[i}\partial_{j]}$, $P_a \sim \partial_a$

Conserved Noether currents:

energy-momentum: $\partial_i \mathfrak{T}_k{}^i = 0$ $\quad \mathfrak{T}^i{}_k := \mathcal{L}\delta^i_k - \frac{\partial \mathcal{L}}{\partial(\partial_i \Phi^A)}\left(\partial_k \Phi^A\right)$

total angular momentum: $\partial_i \left(\mathfrak{S}_{kl}{}^i + x_{[k}\mathfrak{T}_{l]}{}^i\right) = 0$ $\quad \mathfrak{S}_{kl}{}^i := \frac{\partial \mathcal{L}}{\partial(\partial_i \Phi^A)} f_{[kl]}{}^A{}_B \Phi^B$

Note: the semidirect product structure is everywhere.

Gauging the Poincaré group (1/2)

Starting point: a field theory in Minkowski space, $\mathcal{L} = \mathcal{L}(\phi^{\mathsf{A}}, \partial\phi^{\mathsf{A}})$

How do we implement the active Poincaré transformations of the matter field?

Gauging the Poincaré group (1/2)

Starting point: a field theory in Minkowski space, $\mathcal{L} = \mathcal{L}(\phi^A, \partial\phi^A)$

How do we implement the active Poincaré transformations of the matter field?

- we **cannot** simply use $x'^i \mapsto x'^i = \Lambda^i{}_a x^a + \epsilon^i$,
  becomes a diffeomorphism upon localization

Gauging the Poincaré group (1/2)

Starting point: a field theory in Minkowski space, $\mathcal{L} = \mathcal{L}(\phi^A, \partial\phi^A)$

How do we implement the active Poincaré transformations of the matter field?

- we **cannot** simply use $x'^i \mapsto x'^i = \Lambda^i{}_a x^a + \epsilon^i$,
  becomes a diffeomorphism upon localization

- instead, define gauge transformations with respect to anholonomic, coordinate-independent coframe: $\vartheta^\mu = \delta^\mu_a dx^a$

- corresponds to an operational understanding of the Poincaré transformations

Gauging the Poincaré group (1/2)

Starting point: a field theory in Minkowski space, $\mathcal{L} = \mathcal{L}(\phi^A, \partial\phi^A)$

How do we implement the active Poincaré transformations of the matter field?

- we **cannot** simply use $x'^i \mapsto x'^i = \Lambda^i{}_a x^a + \epsilon^i$,
  becomes a diffeomorphism upon localization
- instead, define gauge transformations with respect to anholonomic, coordinate-independent coframe: $\vartheta^\mu = \delta^\mu_a dx^a$
- corresponds to an operational understanding of the Poincaré transformations

Action on the field (generators satisfy Poincaré algebra):

$$\phi^A \mapsto \Pi\phi^A := \left(\delta^A_J + \Lambda^{\alpha\beta} f_{\alpha\beta}{}^A{}_J - \delta^A_J \Lambda_\alpha{}^\beta \delta^\alpha_i x^i \partial_\beta - \delta^A_J \epsilon^\alpha \partial_\alpha\right) \phi^J(x)$$

rotation induced translation

$\rightarrow$ angular momentum

Gauging the Poincaré group (2/2)

Reminder: $\phi^A \mapsto \Pi\phi^A := \left(\delta^A_J + \Lambda^{\alpha\beta} f_{\alpha\beta}{}^A{}_J - \delta^A_J \Lambda_\alpha{}^\beta \delta^\alpha_i x^i \partial_\beta - \delta^A_J \epsilon^\alpha \partial_\alpha\right) \phi^J(x)$

Invariance is spoiled: $\delta S = (\Pi S) - S = \int \left[\left(-\partial_i \Lambda^{\alpha\beta}\right) \mathfrak{S}_{\beta\alpha}{}^i + \left(\partial_i \epsilon^\gamma - \Lambda_\beta{}^\gamma \delta^\beta_i\right) \mathfrak{T}_\gamma{}^i\right] d^4x$

Gauging the Poincaré group (2/2)

Reminder: $\phi^A \mapsto \Pi\phi^A := \left(\delta^A_J + \Lambda^{\alpha\beta} f_{\alpha\beta}{}^A{}_J - \delta^A_J \Lambda_\alpha{}^\beta \delta^\alpha_i x^i \partial_\beta - \delta^A_J \epsilon^\alpha \partial_\alpha\right) \phi^J(x)$

Invariance is spoiled: $\delta S = (\Pi S) - S = \int \left[\left(-\partial_i \Lambda^{\alpha\beta}\right) \mathfrak{S}_{\beta\alpha}{}^i + \left(\partial_i \epsilon^\gamma - \Lambda_\beta{}^\gamma \delta^\beta_i\right) \mathfrak{T}_\gamma{}^i\right] d^4x$

How to rescue invariance? Introduce gauge potentials:

$$\begin{aligned} \delta^i_\mu &\mapsto e^i{}_\mu(x), && = \text{tetrad, 4 translational gauge potentials} \\ \partial_i &\mapsto D_i := \partial_i - \Gamma_i{}^{\alpha\beta}(x) f_{\alpha\beta} && = \text{connection, 6 rotational gauge potentials} \end{aligned}$$

Inhomogeneous transformation of gauge potentials ensures local Poincaré invariance.

Gauging the Poincaré group (2/2)

Reminder: $\phi^A \mapsto \Pi\phi^A := \left(\delta^A_J + \Lambda^{\alpha\beta} f_{\alpha\beta}{}^A{}_J - \delta^A_J \Lambda_\alpha{}^\beta \delta^\alpha_i x^i \partial_\beta - \delta^A_J \epsilon^\alpha \partial_\alpha\right) \phi^J(x)$

Invariance is spoiled: $\delta S = (\Pi S) - S = \int \left[\left(-\partial_i \Lambda^{\alpha\beta}\right) \mathfrak{S}_{\beta\alpha}{}^i + \left(\partial_i \epsilon^\gamma - \Lambda_\beta{}^\gamma \delta^\beta_i\right) \mathfrak{T}_\gamma{}^i\right] d^4x$

How to rescue invariance? Introduce gauge potentials:

$$\begin{aligned} \delta^i_\mu &\mapsto e^i{}_\mu(x), && = \text{tetrad, 4 translational gauge potentials} \\ \partial_i &\mapsto D_i := \partial_i - \Gamma_i{}^{\alpha\beta}(x) f_{\alpha\beta} && = \text{connection, 6 rotational gauge potentials} \end{aligned}$$

Inhomogeneous transformation of gauge potentials ensures local Poincaré invariance.

Gauge field strengths:

$$\begin{aligned} F_{ij}{}^\mu &:= 2\left(\partial_{[i} e_{j]}{}^\mu + \Gamma_{[i}{}^{\mu\alpha} e_{j]}{}^\beta g_{\alpha\beta}\right) && = \text{translational curvature} \\ F_{ij}{}^{\mu\nu} &:= 2\left(\partial_{[i} \Gamma_{j]}{}^{\mu\nu} + \Gamma_{[i}{}^{\alpha\mu} \Gamma_{j]}{}^{\beta\nu} g_{\alpha\beta}\right) && = \text{rotational curvature} \end{aligned}$$

Recognizing Riemann–Cartan geometry

The field strengths $\mathsf{F}_{\mathsf{ij}}{}^{\mu}$ and $\mathsf{F}_{\mathsf{ij}}{}^{\mu\nu}$ as well as the covariant derivative D_{i} allow for a geometric interpretation of the resulting structure: a Riemann–Cartan geometry.

$$\begin{aligned} \mathsf{F}_{\mathsf{ij}}{}^{\mu} &:= \mathsf{T}_{\mathsf{ij}}{}^{\mu} && = \text{torsion} \\ \mathsf{F}_{\mathsf{ij}}{}^{\mu\nu} &:= \mathsf{R}_{\mathsf{ij}}{}^{\mu\nu} && = \text{Riemann–Cartan curvature} \end{aligned}$$

Recognizing Riemann–Cartan geometry

The field strengths $F_{ij}{}^{\mu}$ and $F_{ij}{}^{\mu\nu}$ as well as the covariant derivative D_i allow for a geometric interpretation of the resulting structure: a Riemann–Cartan geometry.

$$
\begin{aligned}
F_{ij}{}^{\mu} &:= T_{ij}{}^{\mu} &&= \text{torsion} \\
F_{ij}{}^{\mu\nu} &:= R_{ij}{}^{\mu\nu} &&= \text{Riemann–Cartan curvature}
\end{aligned}
$$

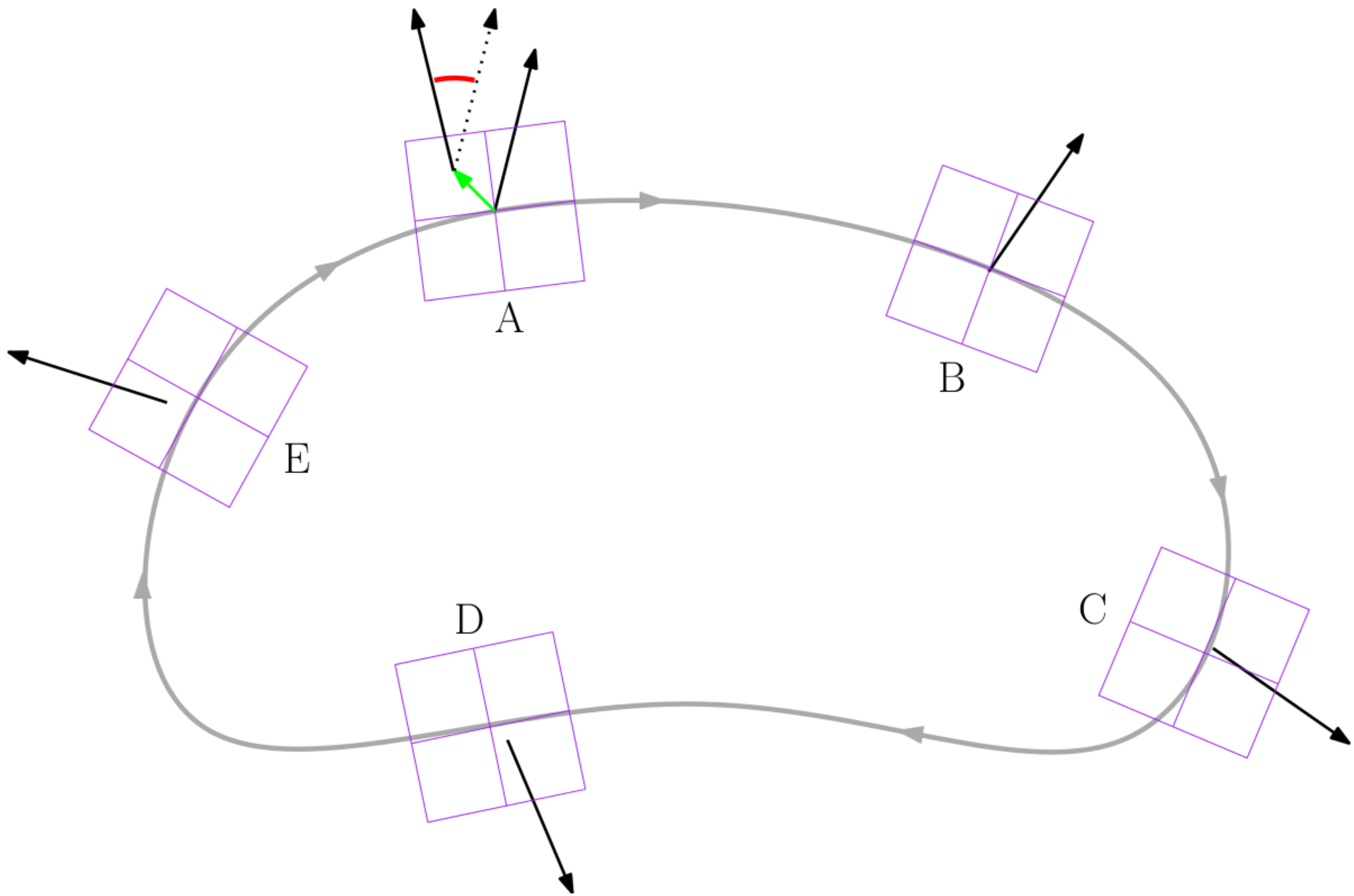
Geometric description in terms of tensor-valued differential forms (Trautman 1973):

$$
\begin{aligned}
\vartheta^{\mu} &= e_a{}^{\mu} dx^i \qquad & T^{\mu} &:= \tfrac{1}{2} T_{ij}{}^{\mu} dx^i \wedge dx^j = d\vartheta^{\mu} + \Gamma^{\mu}{}_{\alpha} \wedge \vartheta^{\alpha} \qquad & DT^{\mu} &= R^{\mu}{}_{\alpha} \wedge \vartheta^{\alpha} \\
\Gamma^{\mu\nu} &= \Gamma_i{}^{\mu\nu} dx^i \qquad & R^{\mu}{}_{\nu} &:= \tfrac{1}{2} R_{ij}{}^{\mu}{}_{\nu} dx^i \wedge dx^j = d\Gamma^{\mu}{}_{\nu} + \Gamma^{\mu}{}_{\alpha} \wedge \Gamma^{\alpha}{}_{\nu} \qquad & DR^{\mu}{}_{\nu} &= 0
\end{aligned}
$$

A more direct interpretation is possible in terms of holonomy.

Geometric interpretation of curvature and torsion

Parallel transport a vector along A $\rightarrow$ B $\rightarrow$ C $\rightarrow$ D $\rightarrow$ E $\rightarrow$ A:



The holonomy of parallel transport is a Poincaré transformation: $\Delta v^{\mu} = A^{\mu}{}_{\alpha} v^{\alpha} \oplus b^{\mu}$

Example: Einstein–Cartan theory (1/2)

The simplest extension of Einstein's General Relativity (Sciama 1960, Kibble 1961).

$$\mathsf{S}_{\mathsf{EC}} = \frac{1}{2\kappa}\int \left(\mathsf{R}^{\alpha\beta}\wedge\eta_{\alpha\beta} - 2\Lambda\eta\right) + \mathcal{L}_{\mathsf{mat}}$$

$$\frac{\delta \mathsf{S}_{\mathsf{EC}}}{\delta\vartheta^{\mu}} = 0 \quad\Rightarrow\quad \frac{1}{2}\eta_{\mu\alpha\beta}\wedge\mathsf{R}^{\alpha\beta} + \Lambda\eta_{\mu} = \kappa\mathfrak{T}_{\mu}$$

$$\frac{\delta \mathsf{S}_{\mathsf{EC}}}{\delta\Gamma^{\mu\nu}} = 0 \quad\Rightarrow\quad \frac{1}{2}\eta_{\mu\nu\alpha}\wedge\mathsf{T}^{\alpha} = \kappa\mathfrak{S}_{\mu\nu}$$

Example: Einstein–Cartan theory (1/2)

The simplest extension of Einstein's General Relativity (Sciama 1960, Kibble 1961).

$$S_{EC} = \frac{1}{2\kappa} \int \left(R^{\alpha\beta} \wedge \eta_{\alpha\beta} - 2\Lambda\eta \right) + \mathcal{L}_{mat}$$

$$\frac{\delta S_{EC}}{\delta\vartheta^{\mu}} = 0 \quad \Rightarrow \quad \frac{1}{2}\eta_{\mu\alpha\beta} \wedge R^{\alpha\beta} + \Lambda\eta_{\mu} = \kappa\mathfrak{T}_{\mu}$$

$$\frac{\delta S_{EC}}{\delta\Gamma^{\mu\nu}} = 0 \quad \Rightarrow \quad \frac{1}{2}\eta_{\mu\nu\alpha} \wedge T^{\alpha} = \kappa\mathfrak{S}_{\mu\nu}$$

The source of curvature is the canonical energy-momentum $\mathfrak{T}_{\mu} = \delta\mathcal{L}_{mat}/\delta\vartheta^{\mu}$,
and torsion is linked algebraically to the spin angular momentum $\mathfrak{S}_{\mu\nu} = \delta\mathcal{L}_{mat}/\delta\Gamma^{\mu\nu}$.

Note that energy-momentum and spin angular momentum are no longer conserved.

Example: Einstein–Cartan theory (2/2)

Rewrite field equations: Substitute spin for torsion and split off post-Riemannian part. Relate to Einstein equations by splitting energy momentum into symmetric energy-momentum $\Theta_{ij} = \Theta_{ji} = \delta\mathcal{L}_{mat}/\delta g^{ij}$ and find $\mathfrak{T}_{ij} = \Theta_{ij} + \left(\kappa\mathfrak{S}^2\right)_{ij}$.

$\rightarrow$ Einstein–Cartan $=$ General Relativity $+$ weak spin-spin contact interaction

Example: Einstein–Cartan theory (2/2)

Rewrite field equations: Substitute spin for torsion and split off post-Riemannian part. Relate to Einstein equations by splitting energy momentum into symmetric energy-momentum $\Theta_{ij} = \Theta_{ji} = \delta\mathcal{L}_{mat}/\delta g^{ij}$ and find $\mathfrak{T}_{ij} = \Theta_{ij} + \left(\kappa\mathfrak{S}^2\right)_{ij}$.

$\rightarrow$ Einstein–Cartan $=$ General Relativity $+$ weak spin-spin contact interaction

When does spin-spin interaction balance the gravitational collapse?

mass density of spins $\approx$ κ (spin density)^2

$\rightarrow$ $r_{EC} \approx \left(\lambda_{Compton} \times \ell^2_{Planck}\right)^{1/3} \approx 10^{-26}$cm $\gg$ $\ell_{Planck} \approx 10^{-33}$cm

Example: Einstein–Cartan theory (2/2)

Rewrite field equations: Substitute spin for torsion and split off post-Riemannian part. Relate to Einstein equations by splitting energy momentum into symmetric energy-momentum $\Theta_{ij} = \Theta_{ji} = \delta\mathcal{L}_{mat}/\delta g^{ij}$ and find $\mathfrak{T}_{ij} = \Theta_{ij} + \left(\kappa\mathfrak{S}^2\right)_{ij}$.

$\rightarrow$ Einstein–Cartan $=$ General Relativity $+$ weak spin-spin contact interaction

When does spin-spin interaction balance the gravitational collapse?

mass density of spins $\approx$ κ (spin density)^2

$\rightarrow$ $r_{EC} \approx \left(\lambda_{Compton} \times \ell^2_{Planck}\right)^{1/3} \approx 10^{-26}\text{cm} \gg \ell_{Planck} \approx 10^{-33}\text{cm}$

Applications:

- contact interaction affects energy-momentum and hence singularity theorems
- bouncing cosmologies (Poplawski 2012; Magueijo, Zlosnik, Kibble 2013; ...)

The deformed Lie algebra in Poincaré gauge theory (1/2)

Poincaré gauge theory is an external gauge theory, unlike Yang–Mills.
Therefore, the localized symmetries affect the Lie algebra of the gauge group.

$$
\begin{aligned}
[M_{\mu\nu}, M_{\alpha\beta}] &= g_{\alpha[\mu} M_{\nu]\beta} - g_{\beta[\mu} M_{\nu]\alpha}, \\
[M_{\mu\nu}, D_{\alpha}] &= g_{\alpha[\mu} D_{\nu]}, \\
[D_{\mu}, D_{\alpha}] &= F_{\mu\alpha}{}^{\rho\sigma} M_{\rho\sigma} - F_{\mu\alpha}{}^{\sigma} D_{\sigma} \neq 0.
\end{aligned}
$$

Is this algebra closed?

$$
\left(\overset{2}{\Lambda}(x), \overset{2}{\epsilon}(x)\right) \circ \left(\overset{1}{\Lambda}(x), \overset{1}{\epsilon}(x)\right) \overset{?}{=} \left(\overset{3}{\Lambda}(x), \overset{3}{\epsilon}(x)\right)
$$

Yes, see Hehl (1979). Closure relations:

$$
\begin{aligned}
\overset{3}{\epsilon}{}^{\alpha} &= -\overset{2}{\Lambda}_{\gamma}{}^{\alpha} \overset{1}{\epsilon}{}^{\gamma} + \overset{1}{\Lambda}_{\gamma}{}^{\alpha} \overset{2}{\epsilon}{}^{\gamma} - \overset{2}{\epsilon}{}^{\beta} \overset{1}{\epsilon}{}^{\gamma} F_{\beta\gamma}{}^{\alpha}, \\
\overset{3}{\Lambda}_{\alpha}{}^{\beta} &= -\overset{2}{\Lambda}_{\alpha}{}^{\gamma} \overset{1}{\Lambda}_{\gamma}{}^{\beta} + \overset{1}{\Lambda}_{\alpha}{}^{\gamma} \overset{2}{\Lambda}_{\gamma}{}^{\beta} - \overset{2}{\epsilon}{}^{\gamma} \overset{1}{\epsilon}{}^{\delta} F_{\gamma\delta\alpha}{}^{\beta}
\end{aligned}
$$

The deformed Lie algebra in Poincaré gauge theory (2/2)

Poincaré gauge theory is an external gauge theory, unlike Yang–Mills.
Therefore, the localized symmetries affect the Lie algebra of the gauge group.

$$\begin{aligned}
[M_{\mu\nu}, M_{\alpha\beta}] &= g_{\alpha[\mu} M_{\nu]\beta} - g_{\beta[\mu} M_{\nu]\alpha}, \\
[M_{\mu\nu}, D_\alpha] &= g_{\alpha[\mu} D_{\nu]}, \\
[D_\mu, D_\alpha] &= F_{\mu\alpha}{}^{\rho\sigma} M_{\rho\sigma} - F_{\mu\alpha}{}^{\sigma} D_\sigma \neq 0.
\end{aligned}$$

Open questions:

- Is it still possible to perform a mass-spin classification à la Wigner (1939)?
- Is there a relation to the deformed Poincaré algebra in relative locality?

Thank you for your attention.

References

[1] F. W. Hehl, P. von der Heyde, G. D. Kerlick and J. M. Nester, "General Relativity with Spin and Torsion: Foundations and Prospects," Rev. Mod. Phys. **48** (1976) 393.

[2] F. W. Hehl, “Four Lectures on Poincaré Gauge Field Theory,” Erice, 1979.

Classical aspects of Poincaré gauge theory of gravity

I will briefly highlight a few cornerstones in the development of gauge theory, and then proceed to the gauge structure present in gravity. Following [1], I will argue that if one wishes to take the fermionic character of matter into account, the Poincaré group will give rise to a satisfactory gauge-theoretical description of gravity. This will include both energy-momentum and spin-angular momentum as sources of the gravitational field.

In a second step, I will elaborate on the emerging structure of a Riemann–Cartan geometry. Einstein–Cartan theory will be sketched, a minimal and viable gauge-theoretical extension of Einstein's General Relativity. If time permits, I will briefly mention its implications for cosmology and the possible resolution of singularities.

I will close by pointing out the deformed Lie algebra of the Poincare group as a result of the gauging procedure: unlike in Yang–Mills theory with its internal symmetry groups, here the Lie algebra is deformed due to the presence of curvature and torsion. The implications of this deformation, both in the classical and quantum regime, remain to be seen.