Report on the symposium of July 23, 2021

The “PhD/Early Postdoc Symposium on Non-locality” is a strategic long-term effort to foster communication and exchange of ideas between early-career scientists. You can find out more about us on our website www.spintwo.net/Symposium/. This is the report on the second symposium, which took place on July 23, 2021.

Suggested program

1. Non-locality in the ultraviolet

- Stability of form factors under renormalization
- Beta functions in non-local theories and their interpretation

2. Non-locality from auxiliary fields

- Lee-Wick theories and their non-local limit point
- Effective field theories

3. Wavelet approach

- General idea of wavelets in physics
- Advantages of wavelets in non-local physics

Attending participants

- Fayez Abu-Ajamieh (postdoc, Centre for High Energy Physics, India)
- Jens Boos (postdoc, William & Mary, United States)
- Anish Ghoshal (postdoc, INFN Tor Vergata Rome, Italy)
- Ivan Kolar (postdoc, Van Swinderen Institute, University of Groningen, Netherlands)
- Yuichi Miyashita (master student, Tokyo Institute of Technology, Japan)

Jens opened the meeting and reminded the participants of the shared goal: to create an open and welcoming atmosphere for early-career scientists working in fields involving aspects of non-locality. After a brief icebreaker the main discussion session was opened.

1 Non-locality in the ultraviolet

1.1 Scattering amplitudes

Anish pointed out that in a recent paper, Modesto and Calcagni [1] studied tree-level scattering amplitudes in non-local field theories which are sometimes plagued with additional divergences due to vertex factors diverging with external Euclidean momentum. These divergences are new and due to non-locality, which is why in the literature it is considered a problem. In their paper, Modesto and Calcagni prove that these divergences do *not* occur in special classes of non-local theories.

1.2 Stability of form factors under renormalization

One of the arguments in favor of infinite-derivative field theories lies in the fact that counter-term renormalization induces all terms that are consistent with the symmetries of the theory, and that includes derivative terms of arbitrary power. However, in the non-local theory all these infinitely many derivatives have a fixed relative factor because they show up via one form factor. A natural question to ask is hence whether the form factor is preserved under renormalization.

Anish mentioned that if one is interested in models with small non-locality that lead only to deviations from the standard local phenomenology then this is well understood. In a paper by Biswas and Okada [2] the Feynman rules for non-local QED are derived by expanding the form factor. Then, in Euclidean space, it becomes clear that higher-order terms are suppressed by the non-local scale and in this Euclidean setting are irrelevant operators. Hence, in this limit, the interpretation of β-functions remains unchanged. The specific example under consideration was the 1-photon-2-fermion vertex. Jens pointed out that the considerations are presented at tree-level. However, since the exponentiated covariant derivative generates n-photon-2-fermion vertices, for all n, one could imagine the 3-photon-2-fermion vertex with two of the photon lines contracted. In order words, could this 1-loop effect in principle generate new, unsuppressed terms that would affect the renormalized structure of the theory? At present this seems to be not understood. Anish also pointed out that in the non-Abelian case the story might be quite different due to the photon self-interaction.

1.3 Beta functions in non-local theories and their interpretation

The usual computation for beta functions at leading order in non-local theories results in the same expression as in the local theory, but with a multiplicative factor that is exponentially suppressed for Euclidean energy scales that are much larger than the non-local scale [3, 4]. This is quite a strong departure from local quantum field theories since it implies that beyond a certain scale all of these theories are asymptotically safe. To this, Anish pointed out that the role of beta functions beyond the non-local scale is not well understood. In other words, is it reasonable to assume that the renormalization scale could ever be above the non-local scale?

Jens asked what a typical expectation for the value of a hypothetical non-local scale could be. Is it clear that it needs to be close to the Planck scale at 10^{19} GeV, or could it be much lower? Anish pointed out that the answer to this question also depends on your assumption as to whether the non-local scale is a constant of Nature or perhaps an emergent quantity. After all, it is not a coupling constant in the usual sense. An example for that, Anish continued, are certain types of string theories considered by Gasperini and Veneziano [Q: reference?] where the string scale does not run, but rather shows up as a fundamental parameter in the four-dimensional theory.

Fayez pointed out that if the non-local scale is to be understood as an imprint of the quantum fuzziness at some fundamental scale then it may very well be a universal property of Nature. For example, it could be the fundamental string scale. It could also be related to a fundamental length scale in the structure of spacetime.

Jens raised the question whether this picture would be consistent from a Wilsonian renormalization perspective, wherein integrating out high momentum modes and insisting on the same form of the

Lagrangian gives rise to the renormalization group flow. Fayez mentioned that it would be hard to imagine that integrating out a single field could ever give rise to an exponential (or, in principle, any other entire function with infinitely many terms). In that context, Fayez asked whether there non-local form factors are typically predicted or inserted by hand.

Jens mentioned the work by Frampton and Okada [5], where a non-local form factor appears in the effective scalar action of a p-adic string, but in most of the literature the form factors are not predicted but inserted by hand, in the hope that they reproduce the desirable features of non-local theories (often: finite and not extra particles, "ghost-free"). Anish added that recently, Luca wrote a paper [6] where the concept of reciprocity (i.e. going from x to $1/x$) is used to determine non-local properties of a theory, based on ideas of Max Born and others. This could perhaps serve as a principle to yield form factors. From personal experience, Anish recalled a conversation from a 2018 conference in Corfu where Paul Frampton and Holger Nielsen talked about mimicking more fundamental theories by toy models (e.g. pion-pion scattering has certain non-local aspects). In that spirit one should probably also understand the somewhat ad hoc form factors introduced in various non-local frameworks.

2 Non-locality from auxiliary fields

Mirroring Fayez' earlier comment on integrating out auxiliary fields, Jens mentioned a recent paper where it was possible to derive non-local field theories by integrating out infinitely many fields [7]. This is similar in spirit to what Gia Dvali [Q: reference?] and others had mentioned at the "Quantum gravity, higher derivatives and non-locality," workshop [8] a few months prior. In a special limit, the model mimicks aspects of Calcagni's non-local diffusion model [9]. Ivan asked whether this model allows for the same interpretation in terms of a heat equation boundary value problem.

Using this auxiliary model, Jens asked if there was any reason to still assume that the scale of non-locality would not run. In other words, if it arises from taking the effective field theory limit of a more fundamental theory, and if it runs in that original theory, then shouldn't the scale also run in the effective theory? Ivan asked to clarify whether the role of the auxiliary fields is really auxiliary, because only if one treated them as physical one could answer Jens' question using this method. Turning this perspective around, Ivan pointed out that it is equally possible to start with a fundamentally non-local theory and recast it as the limiting point of this auxiliary theory.

As a general comment, equations of the sort $\Box\phi = \chi$ can be inverted via $\phi = (1/\Box)\chi$, where $1/\Box$ is a non-local factor, such as in the Deser–Woodard model of non-local cosmology [10]. But of course $1/\Box$ is also the Green's function/2-point function (loosely speaking). However, expressions like $1/\Box$ are not unique because of the zero modes of the $\Box$-operator. As Jens pointed out, these zero modes are *not* relevant for certain classes of non-local models (such as "ghost-free" theories), so they might in principle be used to distinguish between different non-local models.

Anish asked whether it could in principle be possible to get rid of the auxiliary fields, and instead arrive at a non-local theory purely from "real" physical fields; you would still need an infinite number of such fields, but perhaps they exist somewhere in Nature to a good approximation. One example could be some particles in the dark sector; a perhaps more promising avenue would be the string axiverse [11] since axions couple to scalars with a derivative term. However, at present it seems unclear if this can be done.

Jens mentioned that Lee–Wick theories might also serve as a theory with some emergent non-local features, to which Anish added that one needs to make sure that the transition between Minkowski spacetime and Euclidean space is well-defined. Of course this is similar in spirit to the difficulty of performing analytic continuation in the presence of entire function form factors that have essential singularities in certain directions of the complex momentum plane. Anish added that another theoretical model where nonlocality plays a role are Nambu–Jona-Lasinio models of chiral fermions, whose effective action contains non-local factors.

3 Wavelet approach

Due to time constraints the participants decided to dedicate a separate symposium to the wavelet methods in the future, the precise date to be determined.

4 Closing remarks

In the closing remarks it was highlighted that participation from postdocs/graduate students in adjoining fields could be both beneficial for them and to the structure of the symposium. The participants agreed to contact other postdocs/graduate students who could be potentially interested. Moreover, it was suggested to add a dedicated section to the end of future symposia where the participants can discuss and brainstorm possible collaborative ideas. After two hours of discussion, Jens closed the meeting, with the date of the next symposium to be determined (although likely some time in September after the summer break).

Acknowledgements

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References

[1] L. Modesto and G. Calcagni, "Tree-level scattering amplitudes in non-local field theories," 2107.04558 [hep-th].

[2] T. Biswas and N. Okada, "Towards LHC physics with non-local Standard Model," Nucl. Phys. B **898**, 113-131 (2015), 1407.3331 [hep-ph].

[3] A. Ghoshal, A. Mazumdar, N. Okada and D. Villalba, "Stability of infinite-derivative Abelian Higgs models," Phys. Rev. D **97**, no. 7, 076011 (2018), 1709.09222 [hep-th].

[4] A. Ghoshal, A. Mazumdar, N. Okada and D. Villalba, "Non-local non-Abelian gauge theory: Conformal invariance and β-function," Phys. Rev. D **104**, no. 1, 015003 (2021) 2010.15919 [hep-ph].

[5] P. H. Frampton and Y. Okada, "Effective scalar field theory of p-adic string," Phys. Rev. D **37**, 3077-3079 (1988).

[6] L. Buoninfante, "Maximal acceleration, reciprocity & non-locality," 2105.08167 [hep-th].

[7] J. Boos and C. D. Carone, "Asymptotic non-locality," Phys. Rev. D **104**, no. 1, 015028 (2021), 2104.11195 [hep-th].

[8] "Quantum gravity, higher derivatives and non-locality," international workshop https://www.qgrav2021.com/.

[9] G. Calcagni and G. Nardelli, "Non-local gravity and the diffusion equation," Phys. Rev. D **82**, 123518 (2010) 1004.5144 [hep-th].

[10] S. Deser and R. P. Woodard, "Non-local Cosmology," Phys. Rev. Lett. **99**, 111301 (2007), 0706.2151 [astro-ph].

[11] A. Arvanitaki, S. Dimopoulos, S. Dubovsky, N. Kaloper and J. March-Russell, "String Axiverse," Phys. Rev. D **81**, 123530 (2010), 0905.4720 [hep-th].