PHYS 581 - DIFFERENTIAL GEOMETRY FOR PHYSICISTS - LECTURE #0

Sep 3, 2021

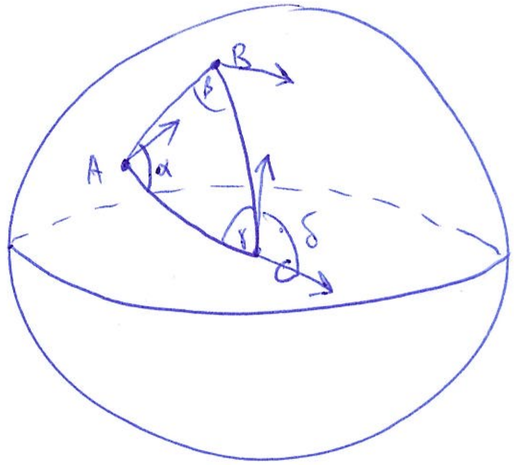
Main topics:

- vectors, matrices, tensors, coordinates
- why you need a covariant derivative
- what is a metric
- geometry of 2D surfaces
- differentiable manifolds
- what is curvature
- why are differential forms so useful

Advanced topics:

- spacetime as a differentiable manifold
- Maxwell's equations in differential forms
- curvature in General Relativity
- black holes
- gauge theories + Lie groups
- holonomy + geometric phases

Example:



$\alpha + \beta + \gamma = ?$ $\quad > 180°$!

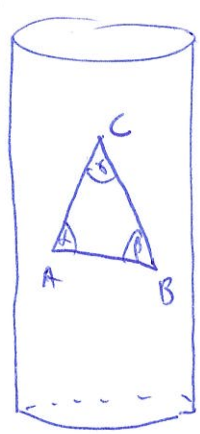
$\delta \neq 0$. ("parallel propagation")

Curved space ($\text{curv} \sim \frac{\text{angle } \delta}{\text{area}}$)

$$\vec{\nabla}^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \varphi^2}$$

why?

What about this:



$\alpha + \beta + \gamma = 180°$!

$\delta = 0$.

But: isn't this also curved?

yes → extrinsic curvature

no → intrinsic curvature

Sneak peek:

vector $\quad \underline{v} = \sum_{i=1}^{n} v^i \hat{e}_i \qquad$ think $|\psi\rangle$, $\begin{pmatrix} \vdots \end{pmatrix}$

covector $\quad \underline{\omega} = \sum_{i=1}^{n} \omega_i \hat{\vartheta}^i \qquad$ think $\langle\phi|$, $(\ldots\ldots)$

interior product $\quad \underline{v} \lrcorner \underline{\omega} = \sum_{\substack{i=1 \\ j=1}}^{n} (v^i \hat{e}_i) \lrcorner (\omega_j \hat{\vartheta}^j) = \sum_{i,j=1}^{n} v^i \omega_j \, \hat{e}_i \lrcorner \hat{\vartheta}^j = \sum_{i,j=1}^{n} v^i \omega_j \, \delta_i^j$

$= \sum_{i=1}^{n} v^i \omega_j \qquad$ think $\langle\phi|\psi\rangle$

matrix $\quad \underline{M} = \sum_{i,j=1}^{n} M^{ij} \, \hat{e}_i \otimes \hat{e}_j \qquad$ think $|\psi\rangle \otimes |\chi\rangle \qquad \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix}$

tensor $\quad \underline{T} = \sum_{i,j,k=1}^{n} T_{ij}{}^{k} \, \hat{\vartheta}^i \otimes \hat{\vartheta}^j \otimes \hat{e}_k \qquad (\, ? \,)$

Some notation:

abstract objects : underline notation, e.g. $\underline{\omega}$ or $\underline{M}$ or $\underline{T}$.

components : ω_i , v^i, M_{ij}, $T_{ij}{}^{k}$

basis : $\hat{e}_i$

cobasis : $\hat{\vartheta}^j$

indices: $i, j, k, l, m, a, b, \ldots$ take values from 1 to n,

n = dimension of space / spacetime / vector space / ...

summation convention : e.g. $\sum_{i=1}^{n} v^i \hat{e}_i = v^i \hat{e}_i$, $T_{ij}{}^{k} \vartheta^i \otimes \vartheta^j \otimes e_k \equiv \sum_{i,j,k=1}^{n} (\ldots)$

$\longrightarrow$ ~~repeat over~~ sum over repeated indices.